NELSON SENIOR MATHS METHODS 12

FULLY WORKED SOLUTIONS

Chapter 6 Applications of integration

Exercise 6.01 Indefinite integrals

Concepts and techniques

1 B $\int 6x^2 dx = 2x^3 + c$

$f(-1) = 2 \Rightarrow c = 4$

$y = 2x^3 + 4$

2 D $\dfrac{dy}{dx} = 4e^{-x}$

$y = \int 4e^{-x} dx = \dfrac{4e^{-x}}{-1} + c$

At $x = 0, y = -9$

$-9 = -4 + c$

$c = -5$

$y = -4e^{-x} - 5$

3 A $f(x) = \int -8 \sin(4x)\, dx = \dfrac{8\cos(4x)}{4} + c = 2\cos(4x) + c$

$-5 = 2\cos\left(\dfrac{4\pi}{4}\right) + c$

$c = -3$

$f(x) = 2\cos(4x) - 3$

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4 **a** $\int x\,dx = \frac{x^2}{2} + c$

b $\int x^2 dx = \frac{x^3}{3} + c$

c $\int x^6 dx = \frac{x^7}{7} + c$

d $\int 2x^4 dx = \frac{2x^5}{5} + c$

e $\int 5x^{-3} dx = \frac{5x^{-2}}{-2} + c = -\frac{5}{2x^2} + c$

f $\int -3x^3 dx = -\frac{3x^4}{4} + c$

g $\int -5x^{-4} dx = -\frac{5x^3}{-3} + c = \frac{5}{3x^3} + c$

h $\int \sqrt{x}\,dx = \frac{2x^{\frac{3}{2}}}{3} + c$

i $\int 5\sqrt{x}\,dx = \frac{10x^{\frac{3}{2}}}{3} + c$

j $\int \frac{x^5}{7} dx = \frac{x^6}{42} + c$

k $\int \frac{x^3}{5} dx = \frac{x^4}{20} + c$

l $\int \frac{x^{-3}}{4} dx = -\frac{x^{-2}}{8} + c = -\frac{1}{8x^2} + c$

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m $\int x^{\frac{1}{3}} dx = \dfrac{3x^{\frac{4}{3}}}{4} + c$

n $\int 3x^{\frac{2}{5}} dx = \dfrac{15x^{\frac{7}{5}}}{7} + c$

o $\int x^{-\frac{3}{4}} dx = 4x^{\frac{1}{4}} + c$

p $\int \dfrac{4}{x^3} dx = -2x^{-2} + c = \dfrac{-2}{x^2} + c$

q $\int \dfrac{-5}{x^6} dx = x^{-5} + c = \dfrac{1}{x^5} + c$

r $\int \dfrac{10}{\sqrt{x}} dx = \int 10x^{-\frac{1}{2}} dx = 20x^{\frac{1}{2}} + c$

s $\int \dfrac{-6}{\sqrt[3]{x}} dx = \int -6x^{-\frac{1}{3}} dx = -9x^{\frac{2}{3}} + c$

t $\int \dfrac{8}{x\sqrt{x}} dx = \int 8x^{-\frac{3}{2}} dx = -16x^{-\frac{1}{2}} + c = -\dfrac{16}{\sqrt{x}} + c$

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5 **a** $\int e^{2x}dx = \frac{e^{2x}}{2} + c$

b $\int e^{4x}dx = \frac{e^{4x}}{4} + c$

c $\int e^{-x}dx = \frac{e^{-x}}{-1} + c = -e^{-x} + c$

d $\int e^{5x}dx = \frac{e^{5x}}{5} + c$

e $\int e^{-2x}dx = \frac{e^{-2x}}{-2} + c = -0.5e^{-2x} + c$

f $\int e^{4x+1}dx = \frac{e^{4x+1}}{4} + c$

g $\int -3e^{5x}dx = \frac{-3e^{5x}}{5} + c$

h $\int e^{2t}dt = \frac{e^{2t}}{2} + c$

i $\int 5e^{4x}dx = \frac{5e^{4x}}{4} + c$

j $\int -6e^{-2x}dx = \frac{-6e^{-2x}}{-2} + c = 3e^{-2x} + c$

k $\int 4e^{\frac{x}{2}}dx = 8e^{\frac{x}{2}} + c$

l $\int 6e^{-\frac{x}{3}}dx = -18e^{-\frac{x}{3}} + c$

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6 **a** $\int \cos(x)\,dx = \sin(x) + c$

b $\int \sin(x)\,dx = -\cos(x) + c$

c $\int \sin(3x)\,dx = -\frac{1}{3}\cos(3x) + c$

d $\int -\sin(7x)\,dx = \frac{1}{7}\cos(7x) + c$

e $\int \cos(x+1)\,dx = \sin(x+1) + c$

f $\int \sin(2x-3)\,dx = -\frac{1}{2}\cos(2x-3) + c$

g $\int \cos(2x-1)\,dx = \frac{\sin(2x-1)}{2} + c$

h $\int 4\sin\left(\frac{x}{2}\right)dx = -8\cos\left(\frac{x}{2}\right) + c$

i $\int -\sin(3-x)\,dx = -\cos(3-x) + c$

j $\int 3\cos\left(\frac{x}{4}\right)dx = 12\sin\left(\frac{x}{4}\right) + c$

k $\int \sin(\pi - x)\,dx = -\frac{\cos(\pi - x)}{-1} = \cos(\pi - x) + c$

l $\int \cos(x+\pi)\,dx = \sin(x+\pi) + c$

m $\int -2\sin\left(\frac{2x}{5}\right)dx = 5\cos\left(\frac{2x}{5}\right) + c$

n $\int 4\cos\left(\frac{7x}{4}\right)dx = \frac{16}{7}\sin\left(\frac{7x}{4}\right) + c$

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o $\int 2\cos\left(\frac{\pi x}{3}\right)dx = \frac{6}{\pi}\sin\left(\frac{\pi x}{3}\right) + c$

p $\int -2\sin\left(\frac{-3x}{\pi}\right)dx = \frac{-2\pi}{-3}\times\left[-\cos\left(\frac{-3x}{\pi}\right)\right] + c = -\frac{2\pi}{3}\cos\left(\frac{-3x}{\pi}\right) + c$

7 **a** $\int (x+1)^4\,dx = \frac{(x+1)^5}{5} + c$

b $\int (5x-1)^9\,dx = \frac{(5x-1)^{10}}{50} + c$

c $\int (3y-2)^7\,dx = \frac{(3y-2)^8}{24} + c$

d $\int (4+3x)^4\,dx = \frac{(4+3x)^5}{15} + c$

e $\int (7x+8)^{12}\,dx = \frac{(7x+8)^{13}}{91} + c$

f $\int (1-x)^6\,dx = \frac{(1-x)^7}{-7} + c = -\frac{(1-x)^7}{7} + c$

g $\int \sqrt{2x-5}\;dx = \int (2x-5)^{\frac{1}{2}}dx = \frac{2}{3}\times\frac{1}{2}(2x-5)^{\frac{3}{2}} + c = \frac{\sqrt{(2x-5)^3}}{3} + c$

h $\int 2(3x+1)^{-4}\,dx = \frac{2(3x+1)^{-3}}{-9} + c = -\frac{2(3x+1)^{-3}}{9} + c$

i $\int 3(x+7)^{-2}\,dx = \frac{3(x+\ 7)^{-1}}{-1} + c = -\frac{3}{x+7} + c$

j $\int \frac{1}{2(4x-5)^3}dx = \int \frac{(4x-5)^{-3}}{2}dx = \frac{(4x-5)^{-2}}{-16} + c = -\frac{1}{16(4x-5)^2} + c$

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k $\int \sqrt[3]{4x+3}\ dx = \int (4x+3)^{\frac{1}{3}} dx = \dfrac{4(4x+3)^{\frac{4}{3}}}{12} + c = \dfrac{3\sqrt[3]{\left(4x+3\right)^4}}{16} + c$

l $\int (2-x)^{-\frac{1}{2}}\, dx = \dfrac{2(2-x)^{\frac{1}{2}}}{-1} + c = -2\sqrt{2-x} + c$

m $\int \sqrt{(t+3)^3}\, dt = \int (t+3)^{\frac{3}{2}} dt = \dfrac{2(t+3)^{\frac{5}{2}}}{5} + c = \dfrac{2\sqrt{(t+3)^5}}{5} + c$

n $\int \sqrt{(5x+2)^5}\, dx = \int (5x+2)^{\frac{5}{2}} dx = \dfrac{2(5x+2)^{\frac{7}{2}}}{7\times 5} + c = \dfrac{2\sqrt{(5x+2)^7}}{35} + c$

o $\int \left(4-5x\right)^{-4} dx = \dfrac{(4-5x)^{-3}}{-3(-5)} + c = \dfrac{1}{15(4-5x)^3} + c$

p $\int -6\left(3-4x\right)^{-5} dx = \dfrac{-6\left(3-4x\right)^{-4}}{-4(-4)} + c = -\dfrac{3}{8\left(3-4x\right)^4} + c$

Reasoning and communication

8 $y = \int -3x\, dx = \dfrac{-3x^2}{2} + c$

$y = \dfrac{-3x^2}{2} + c$ Using $(2,\ 2) \Rightarrow 2 = -6 + c$

$y = \dfrac{-3x^2}{2} + 8$

9 $y = \int 3e^{2x} dx = \dfrac{3e^{2x}}{2} + c$

$y = \dfrac{3e^{2x}}{2} + c$ Using $(0, 5.5) \Rightarrow 4 = c$

$y = \dfrac{3e^{2x}}{2} + 4$

10 $\frac{d}{dx}\left(e^{x^4}\right) = e^{x^4} \times 4x^3 = 4x^3 e^{x^4}$

$\therefore \int 4x^3 e^{x^4}\,dx = e^{x^4} + c$

$\therefore \int 2x^3 e^{x^4}\,dx = \frac{e^{x^4}}{2} + c$

11 $\frac{d}{dx}\left(4x^2+1\right)^3 = 3\left(4x^2+1\right)^2 \times 8x = 24x\left(4x^2+1\right)^2$

$\therefore \int 24x\left(4x^2+1\right)^2\,dx = \left(4x^2+1\right)^3 + c$

$\therefore \int 6x\,(4x^2+1)^2\,dx = \frac{1}{4}\left(4x^2+1\right)^3 + c$

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Exercise 6.02 Properties of indefinite integrals

Concepts and techniques

1 C $\int 2x^2 dx + \int x\,dx + \int 5\,dx$

2 C $\int (4x+3)\,dx = 2x^2 + 3x + c$

$f(1) = 7 \Rightarrow 7 = 2 + 3 + c \Rightarrow c = 2$

$f(x) = 2x^2 + 3x + 2$

3 A $\int (2x^3 - 7x^2)dx$

because $x^2(2x-7) = 2x^3 - 7x^2$

4 D $\frac{dy}{dx} = \frac{ax^2}{3} - x \Rightarrow y = \int\left(\frac{ax^2}{3} - x\right)dx = \frac{ax^3}{9} - \frac{x^2}{2} + c$

$y = \frac{ax^3}{9} - \frac{x^2}{2} + c$ Using (0, 2), $c = 2$

$y = \frac{ax^3}{9} - \frac{x^2}{2} + 2$

5 **a** $\int (m+1)\,dm = \frac{m^2}{2} + m + c$

b $\int (t^2 - 7)\,dt = \frac{t^3}{3} - 7t + c$

c $\int (h^2 + 5)\,dh = \frac{h^3}{3} + 5h + c$

d $\int (y-3)\,dy = \frac{y^2}{2} - 3y + c$

e $\int (2x+4)\,dx = x^2 + 4x + c$

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f $\int (b^2 + b)\,db = \frac{b^3}{3} + \frac{b^2}{2} + c$

g $\int (a^3 - a - 1)\,da = \frac{a^4}{4} - \frac{a^2}{2} - a + c$

h $\int (x^2 + 2x + 5)\,dx = \frac{x^3}{3} + x^2 + 5x + c$

i $\int (4x^3 - 3x^2 + 8x - 1)\,dx = x^4 - x^3 + 4x^2 - x + c$

j $\int (6x^5 + x^4 + 2x^3)\,dx = x^6 + \frac{x^5}{5} + \frac{x^4}{2} + c$

k $\int (x^7 - 3x^6 - 9)\,dx = \frac{x^8}{8} - \frac{3x^7}{7} - 9x + c$

l $\int (2x^3 + x^2 - x - 2)\,dx = \frac{x^4}{2} + \frac{x^3}{3} - \frac{x^2}{2} - 2x + c$

m $\int (x^5 + x^3 + 4)\,dx = \frac{x^6}{6} + \frac{x^4}{4} + 4x + c$

n $\int (4x^2 - 5x - 8)\,dx = \frac{4x^3}{3} - \frac{5x^2}{2} - 8x + c$

o $\int (3x^4 - 2x^3 + x)\,dx = \frac{3x^5}{5} - \frac{x^4}{2} + \frac{x^2}{2} + c$

p $\int (6x^3 + 5x^2 - 4)\,dx = \frac{3x^4}{2} + \frac{5x^3}{3} - 4x + c$

q $\int (3x^{-4} + x^{-3} + 2x^{-2})\,dx = \frac{3x^{-3}}{-3} + \frac{x^{-2}}{-2} + \frac{2x^{-1}}{-1} + c = -\frac{1}{x^3} - \frac{1}{2x^2} - \frac{2}{x} + c$

r $\int(7x^{\frac{3}{2}}-4x+6x^{-\frac{1}{3}})dx=\frac{2}{5}\times 7x^{\frac{5}{2}}-2x^2+\frac{3}{2}\times 6x^{\frac{2}{3}}=\frac{14x^{\frac{5}{2}}}{5}-2x^2+9x^{\frac{2}{3}}+c$

6 **a** $\int\frac{x^6-3x^5+2x^4}{x^3}dx=\int(x^3-3x^2+2x)dx=\frac{x^4}{4}-x^3+x^2+c$

b $\int(1-2x)^2\,dx=\int(4x^2-4x+1)\,dx=\frac{4x^3}{3}-2x^2+x+c$

c $\int(x-2)(x+5)\,dx=\int\left(x^2+3x-10\right)dx=\frac{x^3}{3}+\frac{3x^2}{2}-10x+c$

d $\int\frac{4x^3-x^5-3x^2+7}{x^5}\,dx=\int\,(4x^{-2}-1-3x^{-3}+7x^{-5})\,dx$

$$=\frac{4x^{-1}}{-1}-x-\frac{3x^{-2}}{-2}+\frac{7x^{-4}}{-4}+c$$
$$=-\frac{4}{x}-x+\frac{3}{2x^2}-\frac{7}{4x^4}+c$$

e $\int(y^2-y^{-7}+5)\,dy=\frac{y^3}{3}-\frac{y^{-6}}{-6}+5y+c=\frac{y^3}{3}+\frac{1}{6y^6}+5y+c$

f $\int(t^2-4)(t-1)\,dt=\int(t^3-t^2-4t+4)\,dt=\frac{t^4}{4}-\frac{t^3}{3}-2t^2+4t+c$

g $\int\sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)dx=\int\left(\sqrt{x}+1\right)dx=\frac{2x^{\frac{3}{2}}}{3}+x+c=\frac{2\sqrt{x^3}}{3}+x+c$

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h $\int \frac{(x+5)(x-2)}{x^4}dx = \int \frac{(x^2+3x-10)}{x^4}dx$

$$= \int \left(x^{-2}+3x^{-3}-10x^{-4}\right)dx$$

$$= \frac{x^{-1}}{-1}+\frac{3x^{-2}}{-2}-\frac{10x^{-3}}{-3}+c$$

$$= -\frac{1}{x}-\frac{3}{2x^2}+\frac{10}{3x^3}+c$$

i $\int \frac{2x^2-4x+3}{\sqrt{x}}dx \ = \int (2x^{\frac{3}{2}}-4x^{\frac{1}{2}}+3x^{-\frac{1}{2}})dx$

$$= \frac{2}{5}\times 2x^{\frac{5}{2}}-\frac{2}{3}\times 4x^{\frac{3}{2}}+2\times 3x^{\frac{1}{2}}+c$$

$$= \frac{4x^{\frac{5}{2}}}{5}-\frac{8x^{\frac{3}{2}}}{3}+6x^{\frac{1}{2}}+c$$

$$= \frac{4\sqrt{x^5}}{5}-\frac{8\sqrt{x^3}}{3}+6\sqrt{x}+c$$

7 **a** $\frac{dy}{dx} = 2x - 5$

$y = \int (2x-5)\,dx = x^2 - 5x + c$

$y = x^2 - 5x + c$

$(-1, 8) \Rightarrow 8 = 1 + 5 + c \Rightarrow c = 2$

$y = x^2 - 5x + 2$

b $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 4x$

$y = \int (3x^{\frac{1}{2}} - 4x)\,dx = 2x^{\frac{3}{2}} - 2x^2 + c$

$y = 2x^{\frac{3}{2}} - 2x^2 + c$

$(4, -6) \Rightarrow -6 = 16 - 32 + c \Rightarrow c = 10$

$y = 2x^{\frac{3}{2}} - 2x^2 + 10$

c $\frac{dy}{dx} = 3x^2 - x + 2$

$y = \int (3x^2 - x + 2)\,dx = x^3 - \frac{x^2}{2} + 2x + c$

$y = x^3 - \frac{x^2}{2} + 2x + c$

$(2, 0) \Rightarrow 0 = 8 - 2 + 4 + c \Rightarrow c = -10$

$y = x^3 - \frac{x^2}{2} + 2x - 10$

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8 **a** $f(x) = \int (6x - 1)\,dx$

$f(x) = 3x^2 - x + c$

$(0, 5) \Rightarrow c = 5$

$f(x) = 3x^2 - x + 5$

b $f(x) = \int (7 - 4x)\,dx$

$f(x) = 7x - 2x^2 + c$

$(-1, 1) \Rightarrow 1 = -7 - 2 + c \Rightarrow c = 10$

$f(x) = 7x - 2x^2 + 10$

c $f(x) = \int (3x^{-2} + 2)\,dx$

$f(x) = -3x^{-1} + 2x + c$

$(1, 5) \Rightarrow 5 = -3 + 2 + c \Rightarrow c = 6$

$f(x) = -3x^{-1} + 2x + 6$

d $f(x) = \int \left(\frac{2}{\sqrt{x}} + 3x\right) dx = 4x^{\frac{1}{2}} + \frac{3x^2}{2} + c = 4\sqrt{x} + \frac{3x^2}{2} + c$

$f(1) = 3 \Rightarrow 3 = 4 + 1.5 + c \Rightarrow c = -2.5$

$f(x) = 4\sqrt{x} + \frac{3x^2}{2} - \frac{5}{2}$

e $f(x) = \int \left(x^{\frac{1}{3}} + 6x^2 - 10\right) dx = \frac{3x^{\frac{4}{3}}}{4} + 2x^3 - 10x + c$

$f(1) = -7 \Rightarrow -7 = 0.75 + 2 - 10 + c \Rightarrow c = 0.25$

$f(x) = \frac{3x^{\frac{4}{3}}}{4} + 2x^3 - 10x + \frac{1}{4} = \frac{3\sqrt[3]{x^4}}{4} + 2x^3 - 10x + \frac{1}{4}$

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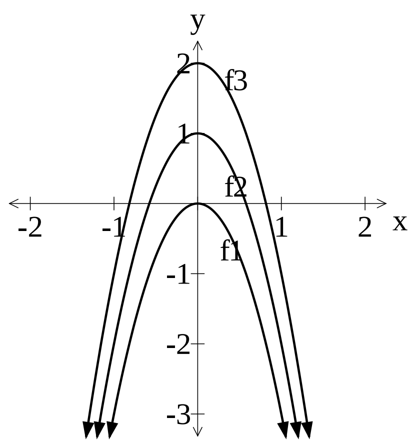
9 **a** $\int -6x\,dx = -3x^2 + c$

b $f_1(x) = -3x^2 + 0$

$f_2(x) = -3x^2 + 1$

$f_3(x) = -3x^2 + 2$

c



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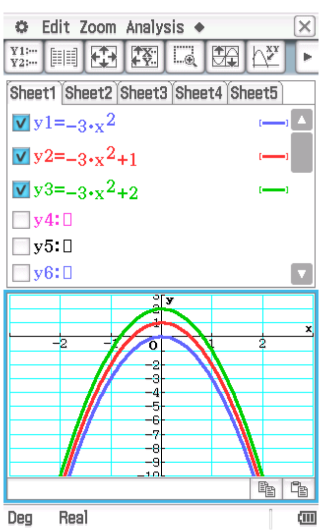
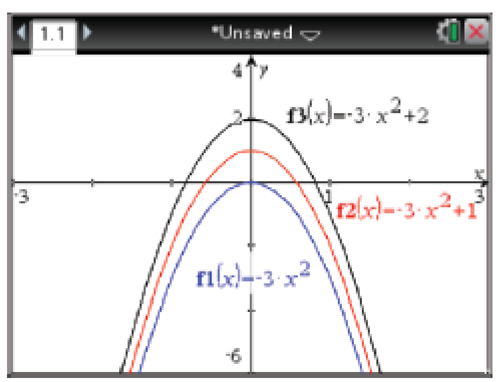


TI-Nspire CAS



The functions are identical but vertically apart by one unit.

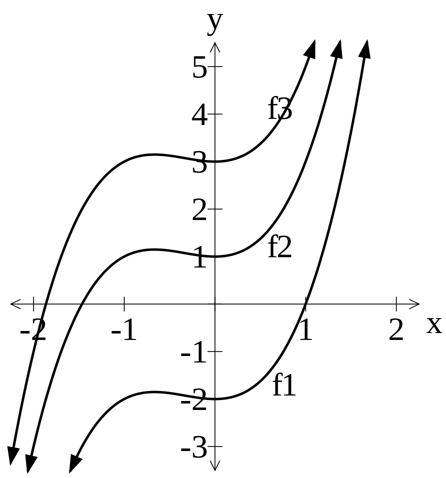
10 **a** $\int (3x^2 + 2x)dx = x^3 + x^2 + c$

b $f_1(x) = x^3 + x^2 - 2$

$f_2(x) = x^3 + x^2 + 1$

$f_3(x) = x^3 + x^2 + 3$

c



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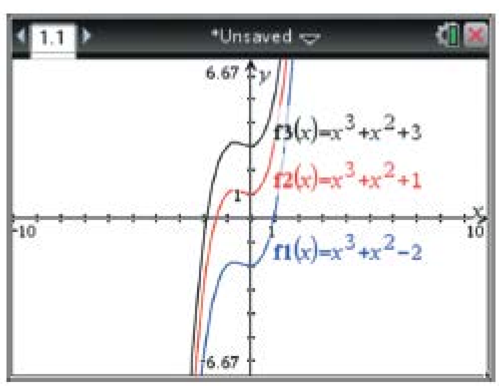

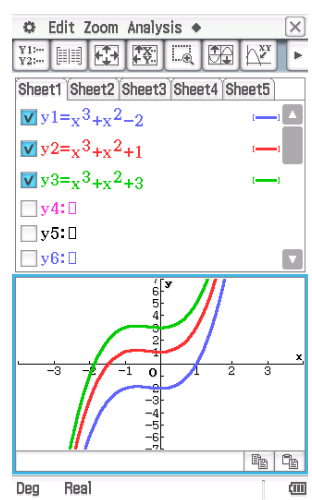

The functions are identical but vertically apart by two and three units.

Reasoning and communication

11 $f'(x)=\frac{3x}{2}+k$

$\frac{3x}{2}+k=0$ at $x=4 \quad \Rightarrow k=-6$

$f(x)=\int\left(\frac{3x}{2}-6\right)dx$

$=\frac{3x^2}{4}-6x+c$

$(4,-2) \quad \Rightarrow -2=12-24+c \Rightarrow c=10$

$f(x)=\frac{3x^2}{4}-6x+10$

$\therefore \quad f(2)=1$

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12 $\dfrac{dy}{dx}=\dfrac{16x-\sqrt{x}}{x^3}=16x^{-2}-x^{-\frac{5}{2}}$

$$y=\int 16x^{-2}-x^{-\frac{5}{2}}dx$$

$$=-16x^{-1}+\frac{2x^{-\frac{3}{2}}}{3}+k$$

$$\left(\frac{1}{4},\ 8\right)\Rightarrow\ 8=-64+\frac{16}{3}+k$$

$k=66\frac{2}{3}$

$$y=-16x^{-1}+\frac{2x^{-\frac{3}{2}}}{3}+66\frac{2}{3}$$

$$=\frac{-16}{x}+\frac{2}{3x^{\frac{3}{2}}}+66\frac{2}{3}$$

$$=\frac{-16}{x}+\frac{2x^{\frac{1}{2}}}{3x^2}+\frac{200}{3}$$

$$=\frac{2\sqrt{x}-48x+200x^2}{3x^2}$$

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Exercise 6.03 Areas under curves

Concepts and techniques

1 **a** $\frac{1}{2}\times 5\times 5 = 12.5\,\text{units}^2$

b $\int_0^5 x\,dx = \left[\frac{x^2}{2}\right]_0^5 = \frac{1}{2}(25-0) = 12.5$

2 **a** $\frac{1}{2}\times 6\times 6 = 18\,\text{units}^2$

b $\int_0^6 6 - x\,dx = \left[6x - \frac{x^2}{2}\right]_0^6 = (18-0) = 18$

3 **a** $\int_{-1}^{2.5} (-2x+5)\,dx$

b $\int_1^5 (x+5)\,dx$

c $\int_{-3}^{-1} (x^2)\,dx$

d $\int_2^4 (2x^2)\,dx$

e $-\int_{-1}^{1} (-e^x)\,dx$

f $-\int_3^5 (x^3 - 7x^2 + 4x + 11)\,dx$

g $\int_0^{\pi} [3\sin(x)]\,dx$

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h $\int_2^8 \left(-x^3 + 10x^2 - 5x\right) dx$

4 **a** $\int_1^{10} (9x+7)\,dx = \left[\frac{9x^2}{2} + 7x\right]_1^{10} = \left(450+70\right) - \left(4.5+7\right) = 508.5$

b $\int_0^6 8\,dx = \left[8x\right]_0^6 = \left(48\right) - \left(0\right) = 48$

c $\int_2^{10} 5x^3 dx = \left[\frac{5x^4}{4}\right]_2^{10} = \left(12\ 500\right) - \left(20\right) = 12\,480$

d $\int_{-3}^{3} x^6 dx = \left[\frac{x^7}{7}\right]_{-3}^{3} = \frac{1}{7}\left(\left[3^7\right] - \left[(-3)^7\right]\right) = 624.6857$

e $\int_0^8 6x^3 dx = \left[\frac{3x^4}{2}\right]_0^8 = 6144 - 0 = 6144$

f $\int_{-5}^{0} (2x^2 - x)dx = \left[\frac{2x^3}{3} - \frac{x^2}{2}\right]_{-5}^{0} = \left(0\right) - \left(-95.8\overline{3}\right) = 95.8\overline{3}$

g $\int_{-12}^{12} (20-m)dm = \left[20m - \frac{m^2}{2}\right]_{-12}^{12} = \left(168\right) - \left(-312\right) = 480$

h $\int_1^2 (4t-7)\,dt = \left[2t^2 - 7t\right]_1^2 = \left(-6\right) - \left(-5\right) = -1$

i $\int_{-3}^{4} (2-x)^2\,dx = \int_{-3}^{4} (4-4x+x^2)\,dx = \left[4x - 2x^2 + \frac{x^3}{3}\right]_{-3}^{4} = \left(5\frac{1}{3}\right) - \left(-39\right) = 44\frac{1}{3}$

j $\int_{-1}^{4} (3x^2 - 2x)\,dx = \left[x^3 - x^2\right]_{-1}^{4} = \left(64-16\right) - (-1-1) = 50$

k $\int_1^3 (4x^2 + 6x - 3)\,dx = \left[\frac{4x^3}{3} + 3x^2 - 3x\right]_1^3 = \left(36+27-9\right) - \left(\frac{4}{3} + 3 - 3\right) = 52\frac{2}{3}$

l $\int_0^1 (x^3 - 3x^2 + 4x)\,dx = \left[\frac{x^4}{4} - x^3 + 2x^2\right]_0^1 = \left(\frac{1}{4} - 1 + 2\right) - (0) = 1\frac{1}{4}$

5 **a** $\int_1^3 \frac{1}{(3x+1)^3}\,dx = \int_1^3 (3x+1)^{-3}\,dx$

$$
\begin{aligned}
&= \left[\frac{(3x+1)^{-2}}{-2\times 3}\right]_1^3 \\
&= -\frac{1}{6}\left[\frac{1}{(3x+1)^2}\right]_1^3 \\
&= -\frac{1}{6}\left(\left(\frac{1}{100}\right) - \left(\frac{1}{16}\right)\right) \\
&= 0.00875
\end{aligned}
$$

b $\int_0^1 \frac{1}{(2x-3)^2}\,dx = \int_0^1 (2x-3)^{-2}\,dx$

$$
\begin{aligned}
&= \left[\frac{(2x-3)^{-1}}{-1\times 2}\right]_0^1 \\
&= \left[\frac{(2x-3)^{-1}}{-2}\right]_0^1 \\
&= \frac{-1}{2}\left[\frac{1}{(2x-3)}\right]_0^1 \\
&= \frac{-1}{2}\left(-1 - \left(-\frac{1}{3}\right)\right) \\
&= \frac{-1}{2} \times \frac{-2}{3} \\
&= \frac{1}{3}
\end{aligned}
$$

c $\int_0^2 \frac{1}{(2x-5)^3}\,dx = \int_0^2 (2x-5)^{-3}\,dx$

$$= \left[\frac{(2x-5)^{-2}}{-4}\right]_0^2$$

$$= \frac{-1}{4}\left[(-1)^{-2} - 25^{-1}\right]$$

$$= \frac{-1}{4}\left(1 - \frac{1}{25}\right)$$

$$= -\frac{6}{25}$$

d $\int_0^1 \frac{3}{(2x+1)^4}\,dx = \int_0^1 3(2x+1)^{-4}\,dx$

$$= \left[\frac{3(2x+1)^{-3}}{-3\times 2}\right]_0^1$$

$$= -\left[\frac{1}{2(2x+1)^3}\right]_0^1$$

$$= -\frac{1}{2}\left(\frac{1}{27} - 1\right)$$

$$= \frac{13}{27}$$

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e $\displaystyle\int_{-1}^{0}\frac{2}{(3x+4)^4}\,dx=\int_{-1}^{0}2(3x+4)^{-4}\,dx$

$$=\left[\frac{2(3x+4)^{-3}}{-3\times 3}\right]_{-1}^{0}$$

$$=-\frac{2}{9}\left[\frac{1}{(3x+4)^3}\right]_{-1}^{0}$$

$$=-\frac{2}{9}\left(\frac{1}{64}-1\right)$$

$$=\frac{2}{9}\times\frac{63}{64}$$

$$=\frac{7}{32}$$

f $\displaystyle\int_{2}^{4}\frac{1}{\sqrt{2x+4}}\,dx=\int_{2}^{4}(2x+4)^{-\frac{1}{2}}\,dx$

$$=\left[\frac{2(2x+4)^{\frac{1}{2}}}{2}\right]_{2}^{4}$$

$$=\left[\sqrt{2x+4}\right]_{2}^{4}$$

$$=\left(\sqrt{12}-\sqrt{8}\right)$$

$$=2\sqrt{3}-2\sqrt{2}$$

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6 **a** $\int_1^3 \frac{1}{x^2}dx = \int_1^3 x^{-2}dx = \left[\frac{x^{-1}}{-1}\right]_1^3 = -\left[\frac{1}{x}\right]_1^3 = 1 - \frac{1}{3} = \frac{2}{3}$

b $\int_1^2 \frac{1}{x^3}dx = \int_1^2 x^{-3}dx = \left[\frac{x^{-2}}{-2}\right]_1^2 = -\frac{1}{2}\left[\frac{1}{x^2}\right]_1^2 = -\frac{1}{2}\left(\frac{1}{4} - 1\right) = \frac{3}{8}$

c $\int_5^{10} 4x^{-2}dx = 4\left[\frac{x^{-1}}{-1}\right]_5^{10} = -4\left[\frac{1}{x}\right]_5^{10} = -4\left(\frac{1}{10} - \frac{1}{5}\right) = \frac{2}{5}$

d $\int_{2.4}^{5.8} \frac{3}{x^4}dx = 3\int_{2.4}^{5.8} x^{-4}dx = 3\left[\frac{x^{-3}}{-3}\right]_{2.4}^{5.8} = -\left[\frac{1}{x^3}\right]_{2.4}^{5.8} = 0.067\,21$

e $\int_2^6 2x^{-3}dx = 2\int_2^6 x^{-3}dx = 2\left[\frac{x^{-2}}{-2}\right]_2^6 = -\left[\frac{1}{x^2}\right]_2^6 = -\left(\frac{1}{36} - \frac{1}{4}\right) = \frac{8}{36} = \frac{2}{9}$

f $\int_1^3 \frac{3x^2 + 2x}{x^4}dx = \int_1^3 3x^{-2} + 2x^{-3}dx = -\left[\frac{3}{x} + \frac{1}{x^2}\right]_1^3 = -\left(\left(1 + \frac{1}{9}\right) - (4)\right) = 2\frac{8}{9}$

7 **a** $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{3x}\,dx = \left[\frac{e^{3x}}{3}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{3}\left(e^{\frac{3\pi}{4}} - e^{-\frac{3\pi}{4}}\right)$

b $\int_3^8 5e^n\,dn = 5\left[e^n\right]_3^8 = 5\left(e^8 - e^3\right) = 5e^3\left(e^5 - 1\right)$

c $\int_0^1 e^{5x}\,dx = \left[\frac{e^{5x}}{5}\right]_0^1 = \frac{1}{5}\left(e^5 - 1\right)$

d $\int_0^2 -e^{-x}\,dx = \left[e^{-x}\right]_0^2 = \left(e^{-2} - 1\right) = \left(\frac{1}{e^2} - 1\right)$

e $\int_1^4 2e^{3x+4}\,dx = 2\left[\frac{e^{3x+4}}{3}\right]_1^4 = \frac{2}{3}\left(e^{16} - e^7\right) = \frac{2}{3}e^7\left(e^9 - 1\right)$

f $\int_2^3 (3x^2 - e^{2x})\,dx = \left[x^3 - \frac{e^{2x}}{2}\right]_2^3 = \left(\left(27 - \frac{e^6}{2}\right) - \left(8 - \frac{e^4}{2}\right)\right) = 19 - \frac{e^6}{2} + \frac{e^4}{2}$

g $\int_0^2 (e^{2x} + 1)\,dx = \left[\frac{e^{2x}}{2} + x\right]_0^2 = \left(\left(\frac{e^4}{2} + 2\right) - \left(\frac{1}{2}\right)\right) = \frac{e^4}{2} + \frac{3}{2}$

h $\int_1^2 (e^x - x)\,dx = \left[e^x - \frac{x^2}{2}\right]_1^2 = e^2 - 2 - \left(e - \frac{1}{2}\right) = e^2 - e - \frac{3}{2}$

i $\int_0^3 (e^{2x} - e^{-x})\,dx = \left[\frac{e^{2x}}{2} + e^{-x}\right]_0^3 = \frac{e^6}{2} + \frac{1}{e^3} - \left(\frac{1}{2} + 1\right) = \frac{e^6}{2} + \frac{1}{e^3} - \frac{3}{2}$

8 **a** $\int_1^4 e^{3V}\,dV = \left[\frac{e^{3V}}{3}\right]_1^4 = \frac{1}{3}\left(e^{12} - e^3\right) = 54\,244.90$

b $\int_1^3 e^{-x}\,dx = -\left[e^{-x}\right]_1^3 = -\left(e^{-3} - e^{-1}\right) = 0.32$

c $\int_0^2 2e^{3y}\,dy = 2\left[\frac{e^{3y}}{3}\right]_0^2 = \frac{2}{3}\left(e^6 - 1\right) = 268.29$

d $\int_5^6 (e^{x+5} + 2x - 3)\,dx = \left[e^{x+5} + x^2 - 3x\right]_5^6$

$= \left(\left(e^{11} + 36 - 18\right) - \left(e^{10} + 25 - 15\right)\right) = e^{11} - e^{10} + 8 = 37\,855.68$

e $\int_0^1 (e^{3t+4} - t)\,dt = \left[\frac{e^{3t+4}}{3} - \frac{t^2}{2}\right]_0^1 = \frac{e^7}{3} - \frac{1}{2} - \left(\frac{e^4}{3}\right) = 346.85$

f $\int_1^2 (e^{4x} + e^{2x})\,dx = \left[\frac{e^{4x}}{4} + \frac{e^{2x}}{2}\right]_1^2 = \frac{e^8}{4} + \frac{e^4}{2} - \left(\frac{e^4}{4} + \frac{e^2}{2}\right) = 755.19$

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9 **a** $\int_0^{\pi} \sin(x)\,dx = -[\cos(x)]_0^{\pi} = -(\cos(\pi) - \cos(0)) = -(-1-1) = 2$

b $\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \cos(2x)\,dx = \left[\frac{\sin(2x)}{2}\right]_{-\frac{\pi}{8}}^{\frac{\pi}{8}} = \frac{1}{2}\left(\sin\left(\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right)\right) = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

c $\int_{\frac{\pi}{2}}^{\pi} \sin\left(\frac{x}{2}\right)dx = -2\left[\cos\left(\frac{x}{2}\right)\right]_{\frac{\pi}{2}}^{\pi} = -2\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{4}\right)\right) = -2\left(0 - \frac{1}{\sqrt{2}}\right) = \sqrt{2}$

d $\int_0^{\frac{\pi}{2}} \cos(3x)\,dx = \left[\frac{\sin(3x)}{3}\right]_0^{\frac{\pi}{2}} = \frac{1}{3}\left(\sin\left(\frac{3\pi}{2}\right) - \sin(0)\right) = -\frac{1}{3}$

e $\int_0^{\frac{1}{2}} \sin(\pi x)\,dx = -\left[\frac{\cos(\pi x)}{\pi}\right]_0^{\frac{1}{2}} = -\frac{1}{\pi}\left(\cos\left(\frac{\pi}{2}\right) - \cos(0)\right) = -\frac{1}{\pi}(0-1) = \frac{1}{\pi}$

f $\int_0^{\frac{\pi}{8}} \sec^2(2x)\,dx = \left[\frac{\tan(2x)}{2}\right]_0^{\frac{\pi}{8}} = \frac{1}{2}\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right) = \frac{1}{2}$

g $\int_0^{\frac{\pi}{12}} 3\cos(2x)\,dx = 3\left[\frac{\sin(2x)}{2}\right]_0^{\frac{\pi}{12}} = \frac{3}{2}\left(\sin\left(\frac{\pi}{6}\right) - \sin(0)\right) = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

h $\int_0^{\frac{\pi}{10}} -\sin(5x)\,dx = \left[\frac{\cos(5x)}{5}\right]_0^{\frac{\pi}{10}} = \frac{1}{5}\left(\cos\left(\frac{\pi}{2}\right) - \cos(0)\right) = \frac{1}{5}(0-1) = -\frac{1}{5}$

10 **a** $\int_2^4 (5t^2+4t+5)\,dt = \left[\frac{5t^3}{3}+2t^2+5t\right]_2^4$

$$= \left(\left(\frac{320}{3}+32+20\right)-\left(\frac{40}{3}+8+10\right)\right) = 127\frac{1}{3}$$

b $\int_0^3 (v^5-4v^3+2v)\,dv = \left[\frac{v^6}{6}-v^4+v^2\right]_0^3 = \left(\left(121.5-81+9\right)-\left(0\right)\right) = 49.5$

c $\int_{-3}^3 (6u^5+5u^4+4)\,du = \left[u^6+u^5+4u\right]_{-3}^3$

$$= \left(\left(729+243+12\right)-\left(729-243-12\right)\right) = 510$$

d $\int_{-1}^1 \frac{72}{(4y+5)^7}\,dy = \int_{-1}^1 72(4y+5)^{-7}\,dy$

$$= \left[\frac{72(4y+5)^{-6}}{-6\times 4}\right]_{-1}^1$$

$$= -3\left[\frac{1}{(4y+5)^6}\right]_{-1}^1$$

$$= -3\left(\frac{1}{9^6}-1\right)$$

$$\approx 3$$

e $\int_1^8 \sqrt[4]{x}\,dx = \frac{4}{5}\left[x^{\frac{5}{4}}\right]_1^8 = 9.96$

f $\int_4^9 \frac{dt}{t^2\sqrt{t}} = \int_4^9 t^{-\frac{5}{2}}\,dt = \frac{-2}{3}\left[t^{-\frac{3}{2}}\right]_4^9 = \frac{-2}{3}\left[\frac{1}{\sqrt{t^3}}\right]_4^9 = -\frac{2}{3}\left(\frac{1}{27}-\frac{1}{8}\right) \approx 0.059$

g $\int_0^2 4e^{2t-3}\,dt = 4\left[\frac{e^{2t-3}}{2}\right]_0^2 = 2\left[e^{2t-3}\right]_0^2 = 2\left(e-e^{-3}\right)$

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h $\int_4^6 \frac{35}{(5h-9)^2}\,dh$

$= 35\int_4^6 (5h-9)^{-2}\,dh = -35\left[\frac{(5h-9)^{-1}}{5}\right]_4^6 = -7\left(\frac{1}{5h-9}\right)_4^6 = -7\left(\frac{1}{21}-\frac{1}{11}\right) = 0.\overline{30}$

i $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 3\sin\left(6x+\frac{\pi}{3}\right)dx$

$= -3\left[\frac{\cos\left(6x+\frac{\pi}{3}\right)}{6}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = -\frac{1}{2}\left(\cos\left(2\pi+\frac{\pi}{3}\right)-\cos\left(-2\pi+\frac{\pi}{3}\right)\right) = -\frac{1}{2}\left(\frac{1}{2}-\frac{1}{2}\right) = 0$

j $\int_4^6 (e^x - x^3)\,dx = \left[e^x - \frac{x^4}{4}\right]_4^6 = \left(e^6 - 324\right) - \left(e^4 - 64\right) = e^6 - e^4 - 260$

k $\int_4^6 \sqrt{4x+1}\,dx = \frac{2}{3}\left[\frac{(4x+1)^{\frac{3}{2}}}{4}\right]_4^6 = \frac{1}{6}\left[\sqrt{(4x+1)^3}\right]_4^6 = 9.15$

l $\int_2^4 16(5-4v)^3\,dv = 16\left[\frac{(5-4v)^4}{4\times(-4)}\right]_2^4 = -(14641-81) = -14560$

m $\int_0^{\frac{\pi}{3}} 6\sin\left(3x-\frac{\pi}{4}\right)dx = -6\left[\frac{\cos\left(3x-\frac{\pi}{4}\right)}{3}\right]_0^{\frac{\pi}{3}}$

$= -2\left(\cos\left(\frac{3\pi}{4}\right)-\cos\left(-\frac{\pi}{4}\right)\right)$

$= -2\left(\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right) = -2\left(\frac{-2}{\sqrt{2}}\right) = 2\sqrt{2}$

n $\int_{-\pi}^{\pi}[\sin(x)-\cos(x)]dx=\left[-\cos(x)-\sin(x)\right]_{-\pi}^{\pi}$

$$=-\left\{\left[\cos(\pi)+\sin(\pi)\right]-\left[\cos\left(-\pi\right)+\sin\left(-\pi\right)\right]\right\}$$
$$=-\left[-1-\left(-1\right)\right]=0$$

o $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{3}{4}\cos\left(\frac{1}{2}x+\frac{\pi}{2}\right)dx$

$$=-2\times\frac{3}{4}\left[\sin\left(\frac{1}{2}x+\frac{\pi}{2}\right)\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}=-\frac{3}{2}\left(\sin\left(\frac{2\pi}{3}\right)-\sin\left(\frac{\pi}{3}\right)\right)=-\frac{3}{2}\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)=0$$

Reasoning and communication

11 **a** $\int_{0}^{3}\frac{1}{x^2}dx$

Cannot be evaluated as $x \neq 0$.

b $\int_{0}^{5}\frac{1}{(x-5)^2}dx$

Cannot be evaluated as $x \neq 5$.

c $\int_{-1}^{3}\frac{1}{(x+1)^3}dx$

Cannot be evaluated as $x \neq -1$.

12 The integral $\int_{0}^{4}\frac{1}{(x-2)^2}dx$ is not valid as there is an undefined point within the bounds.

$x \neq 2$

13 The integral $\int_{-2}^{2} \frac{1}{x} dx$ is not valid as there is an undefined point within the bounds. $x \neq 0$

14 $\frac{d}{dx}\left(xe^{x^2}\right) = 1 \times e^{x^2} + e^{x^2}(2x)(x) = 2x^2e^{x^2} + e^{x^2}$

Therefore $\int_0^1 (2x^2e^{x^2} + e^{x^2})dx = \left[xe^{x^2}\right]_0^1 = e - 0 = e$

15 $V = \int_0^5 5 + 30t^2 dt = \left[5t + 10t^3\right]_0^5 = \left(25 + 1250\right) - 0 = 1275 \text{ m}^3$

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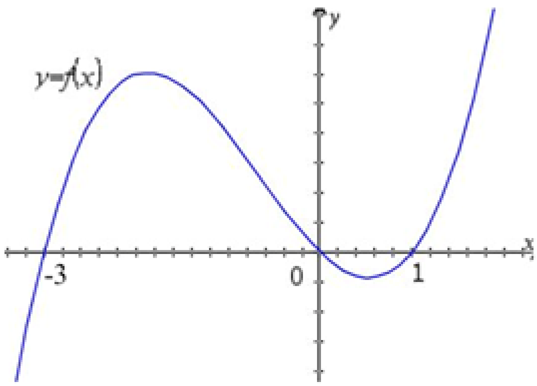
Exercise 6.04 Physical areas

Concepts and techniques

1 D As the *f* values are above the *x*-axis.

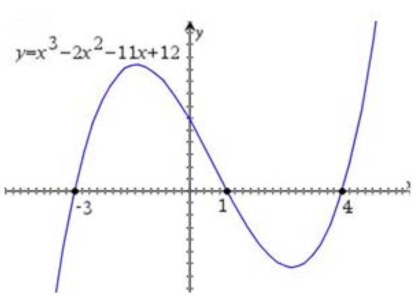


2 B $\int_{-3}^{0} f(x)dx - \int_{0}^{1} f(x)dx$

as $f(x) < 0$ for $0 < x < 1$

3 E

$$\int_{-3}^{1}(x^3 - 2x^2 - 11x + 12)dx$$
$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{11x^2}{2} + 12x\right]_{-3}^{1}$$
$$= 6\tfrac{1}{12} - (-47\tfrac{1}{4}) = 53\tfrac{1}{3}$$



4 D

$$\int_1^4 (x^3 - 2x^2 - 11x + 12)dx$$
$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{11x^2}{2} + 12x\right]_1^4$$
$$= -18\tfrac{2}{3} - 6\tfrac{1}{12} = -24\tfrac{3}{4}$$

5 E $78\tfrac{1}{12}$ using calculator with $\int_{-3}^{4} \text{abs}(x^3 - 2x^2 - 11x + 12)\,dx$

or $\int_{-3}^{1} (x^3 - 2x^2 - 11x + 12)\,dx + \left|\int_1^4 (x^3 - 2x^2 - 11x + 12)\,dx\right|$

6 **a** $\left|\int_0^3 f(x)dx\right| + \int_3^6 f(x)dx$ or $-\int_0^3 f(x)dx + \int_3^6 f(x)dx$

b $\int_{-9}^{-6} g(x)dx - \int_{-6}^{-2} f(x)dx$

c $-\int_{-5}^{4} h(x)dx$

d $\int_{-4}^{-1} k(x)dx - \int_{-1}^{3} k(x)dx$

e $-\int_{-5}^{-2} m(x)dx + \int_{-2}^{2} m(x)dx$

f $\int_1^3 p(x)dx - \int_3^5 p(x)dx + \int_5^6 p(x)dx$

Note: Each of the areas above can be found with the calculator using

$\int_a^b \text{abs}\left(f(x)\right)dx$ where $a \le x \le b$ is the whole interval.

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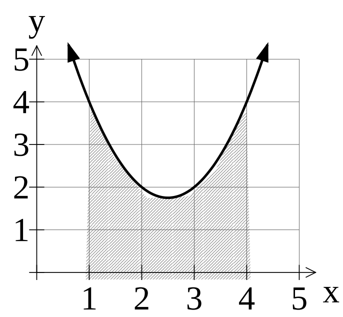
7 **a** $y = x^2 - 5x + 8$ from $x = 1$ to $x = 4$



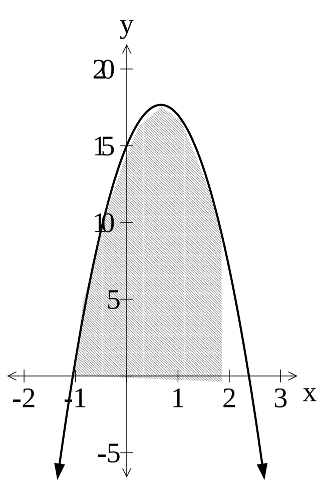
Area:

$$\int_1^4 \left(x^2 - 5x + 8\right) dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} + 8x\right]_1^4$$

$$= \left(\left(\frac{64}{3} - 40 + 32\right) - \left(\frac{1}{3} - \frac{5}{2} + 8\right)\right)$$

$$= 7.5 \text{ units}^2$$

b $f(x) = 15 + 8x - 6x^2$ between $x = -1$ and $x = 2$



$$\int_{-1}^2 \left(15 + 8x - 6x^2\right) dx = \left[15x + 4x^2 - 2x^3\right]_{-1}^2$$

$$= \left((30 + 16 - 16) - (-15 + 4 + 2)\right)$$

$$= 39 \text{ units}^2$$

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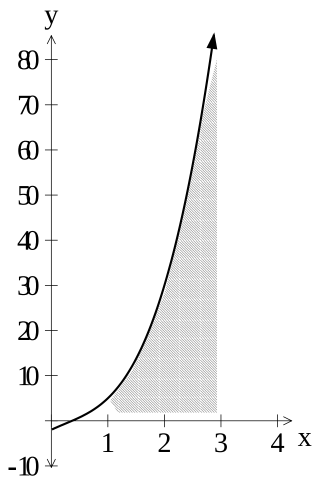
c $y = 4x^3 - 3x^2 + 6x - 2$ between $x = 1$ and $x = 3$



Area:

$$\int_1^3 (4x^3 - 3x^2 + 6x - 2)\,dx = \left[x^4 - x^3 + 3x^2 - 2x\right]_1^3$$
$$= \left(75 - (1)\right)$$
$$= 74 \text{ units}^2$$

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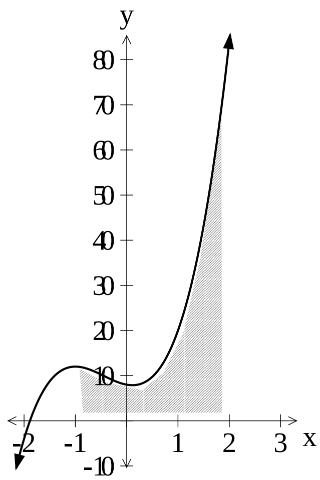
d $f(x) = 6x^3 + 8x^2 - 2x + 8$ from $x = -1$ to $x = 2$



Area:

$$\int_{-1}^{2} (6x^3 + 8x^2 - 2x + 8)\, dx = \left[\frac{3x^4}{2} + \frac{8x^3}{3} - x^2 + 8x\right]_{-1}^{2}$$

$$= \left(57\frac{1}{3} - \left(-10\frac{1}{6}\right)\right)$$

$$= 67.5 \text{ units}^2$$

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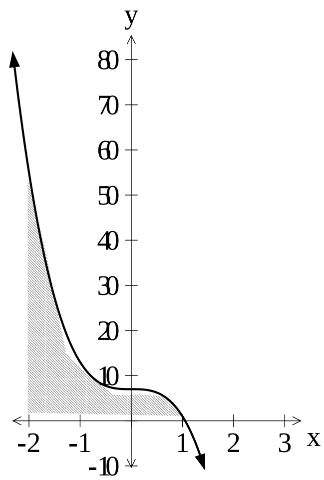
e $y = 7 - 6x^3$ from $x = -2$ to $x = 1$



Area:

$$\int_{-2}^{1} (7 - 6x^3)\, dx = \left[7x - \frac{3x^4}{2} \right]_{-2}^{1}$$

$$= \left(7 - 1.5 - \left(-14 - 24\right)\right)$$

$$= 43.5 \text{ units}^2$$

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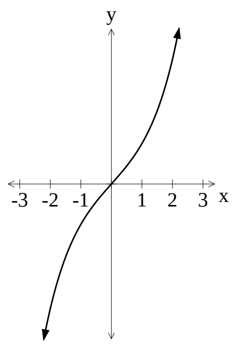
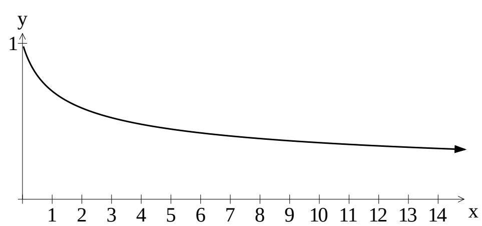
8 $f(x) = e^x - e^{-x}$.



Area:

$$-\int_{-2}^{0}(e^x - e^{-x})\,dx + \int_{0}^{2}(e^x - e^{-x})\,dx = 11.05 \text{ units}^2$$

9 $y = (2x+1)^{-\frac{1}{3}}$



Area:

$$\int_{0}^{13}(2x+1)^{-\frac{1}{3}}dx = 6\,\text{units}^2$$

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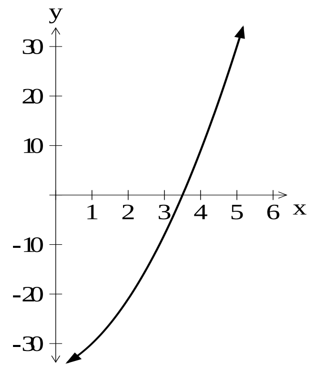
10 **a** $y = 2x^2 + 3x - 35$

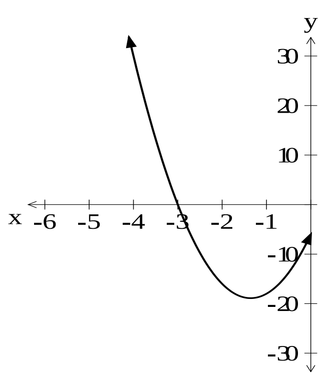


If $y = 0$, $x = ?$, $x = 3.5$

Area between $x = 2$ and $x = 5$ and $y = 0$:

$$-\int_2^{3.5}(2x^2 + 3x - 35)\,dx + \int_{3.5}^5 (2x^2 + 3x - 35)\,dx = 38.25 \text{ units}^2$$

b $y = 7x^2 + 19x - 6$

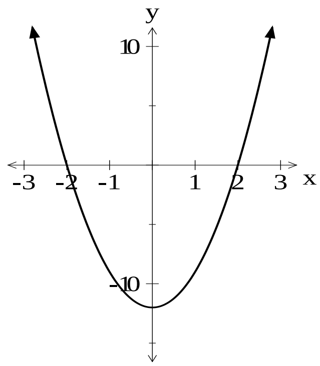


If $y = 0$, $x = ?$, $x = -3$

Area between $x = -5$ and $x = -2$ and the x-axis:

$$\int_{-5}^{-3}(7x^2 + 19x - 6)\,dx - \int_{-3}^{-2}(7x^2 + 19x - 6)\,dx = 73.8\overline{3} \text{ units}^2$$

11 **a** $f(x) = 3x^2 - 12$ and the x-axis



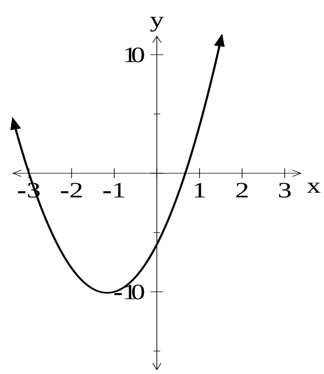
If $y = 0$, $x = ?$, $x = \pm 2$

Area between function and the x-axis:

$$-\int_{-2}^{2} (3x^2 - 12)\, dx = 32\ \text{units}^2$$

Note: This can be calculated using $-2 \times \int_{0}^{2} (3x^2 - 12)\, dx$

b $y = 3x^2 + 7x - 6$ and the x-axis



If $y = 0$, $x = ?$, $x = -3,\ \dfrac{2}{3}$

Area between function and the x-axis:

$$-\int_{-3}^{\frac{2}{3}} (3x^2 + 7x - 6)\, dx = 24.65\ \text{units}^2$$

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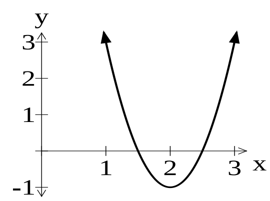
c $f(x) = 4x^2 - 16x + 15$ and the x-axis.

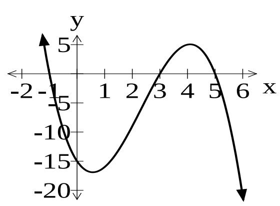


If $y = 0$, $x = ?$, $x = 1.5, 2.5$

Area between function and the x-axis:

$$-\int_{1.5}^{2.5} (4x^2 - 16x + 15)\, dx = \frac{2}{3} \text{ units}^2$$

12 **a** $y = (5 - x)(x + 1)(x - 3)$ by the x-axis



If $y = 0$, $x = ?$, $x = -1, 3, 5$

Area between function and the x-axis:

$$-\int_{-1}^{3} (5 - x)(x + 1)(x - 3)\, dx + \int_{3}^{5} (5 - x)(x + 1)(x - 3)\, dx = 49\frac{1}{3} \text{ units}^2$$

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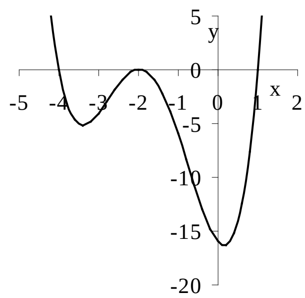
b $y = (x + 2)^2(x - 1)(x + 4)$ by the x-axis.

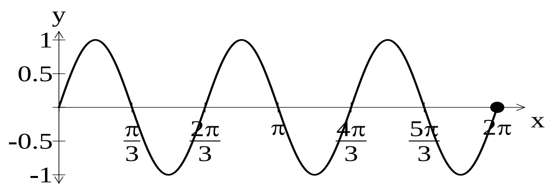


If $y = 0$, $x = ?$, $x = -4, 1$

Area between function and the x-axis:

$$-\int_{-4}^{1} (x+2)^2(x-1)(x+4)\,dx = 31\tfrac{1}{4} \text{ units}^2$$

13 $y = \sin(3x)$ and the x-axis



Area = $6 \times \int_0^{\frac{\pi}{3}} \sin(3x)\,dx = -\frac{6}{3}\left[\cos(3x)\right]_0^{\frac{\pi}{3}} = -2\left(\cos(\pi) - \cos(0)\right) = -2 \times (-2) = 4$ units^2

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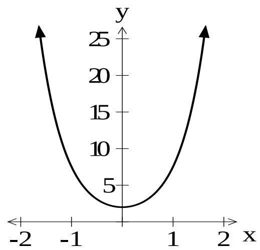
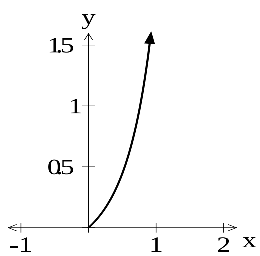
14 $y = e^{2x} + e^{-2x}$, $y = 0$, $x = -1.5$ and $x = 1.5$.



$$\text{Area } = 2\times\int_0^{1.5} e^{2x} + e^{-2x}\,dx = 2\left[\frac{e^{2x}}{2} + \frac{e^{-2x}}{-2}\right]_0^{1.5} = \left[e^{2x} - e^{-2x}\right]_0^{1.5} = e^3 - e^{-3} - (1-1)$$

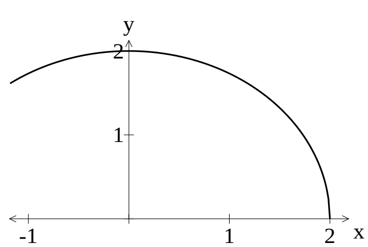
$$= e^3 - e^{-3} \text{ units}^2$$

15 $y = \dfrac{2}{(x-3)^2}$



$$\text{Area } = \int_0^1 \frac{2}{(x-3)^2}\,dx = -2\left[\frac{1}{(x-3)}\right]_0^1 = -2\left(-\frac{1}{2} - \left(\frac{-1}{3}\right)\right) = \frac{1}{3}\text{units}^2$$

16 $y = \sqrt{4 - x^2}$



$$\text{Area } = \int_0^2 \sqrt{4 - x^2}\,dx = \frac{1}{4}\left(\pi \times 2^2\right) = \pi\text{ units}^2$$

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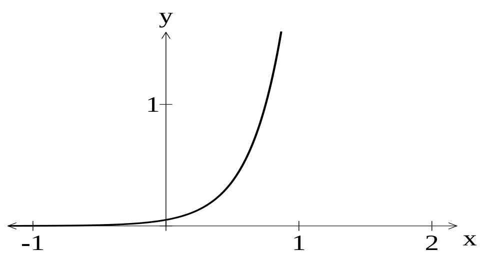
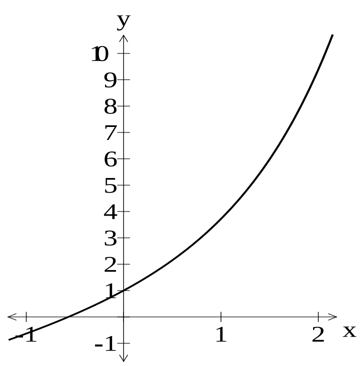
17 $y = e^{4x-3}$



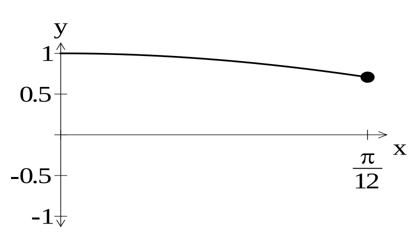
Area = $\int_0^1 e^{4x-3}dx = \left[\frac{e^{4x-3}}{4}\right]_0^1 = \frac{1}{4}\left[e^{4x-3}\right]_0^1 = \frac{1}{4}\left(e - e^{-3}\right)$ units^2

18 $y = x + e^{-x}$



Area = $\int_0^2 x + e^{-x}dx = \left[\frac{x^2}{2} - e^{-x}\right]_0^2 = \left(2 - e^{-2} - \left(0 - e^0\right)\right) = 3 - e^{-2} = 2.86$ (2 d.p.)

19 $y = \cos(3x)$



Area = $\int_0^{\frac{\pi}{12}} \cos(3x)dx = \frac{1}{3}\left[\sin(3x)\right]_0^{\frac{\pi}{12}} = \frac{1}{3}\left(\sin\left(\frac{\pi}{4}\right) - \sin(0)\right) = \frac{1}{3} \times \frac{1}{\sqrt{2}} = \frac{1}{3\sqrt{2}}$

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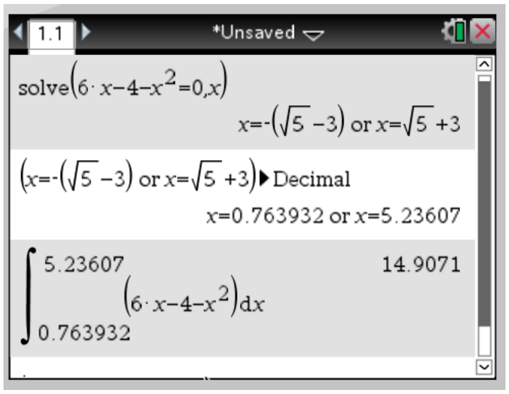
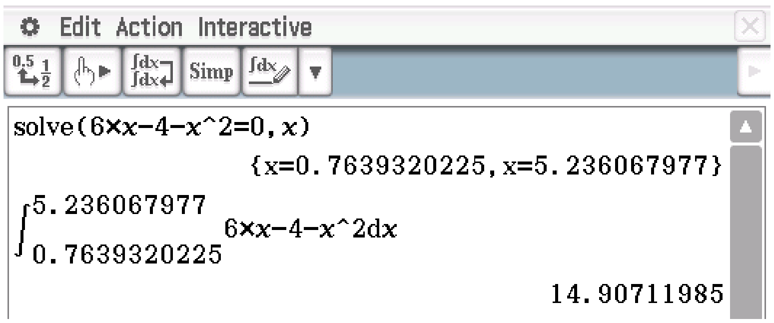
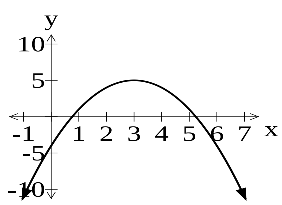
20 **a** TI-Nspire CAS



ClassPad



$y = 6x - 4 - x^2$

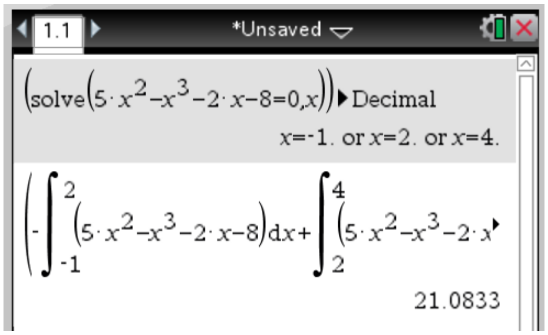


If $y = 0$, $x = ?$, $x = 0.764, 5.236$

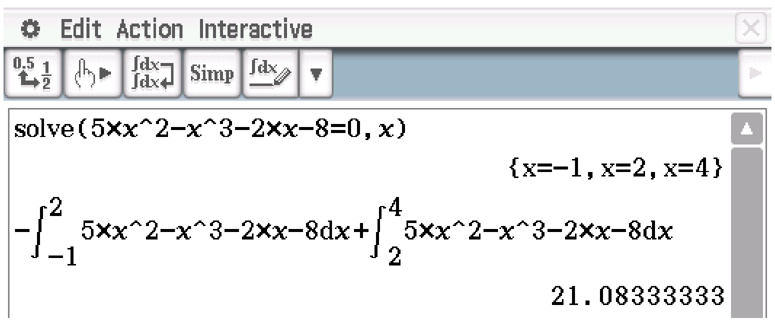
Area between function and the x-axis:

$$\int_{0.764}^{5.236}\left(6x - 4 - x^2\right)dx = 14.91\text{units}^2$$

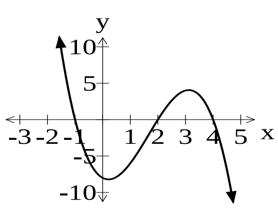
b TI-Nspire CAS



ClassPad



$y = 5x^2 - x^3 - 2x - 8$

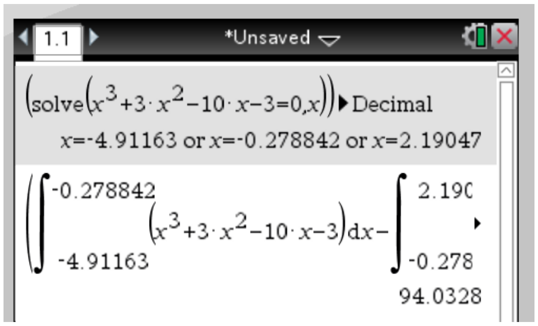


If $y = 0$, $x = ?$, $x = -1, 2, 4$

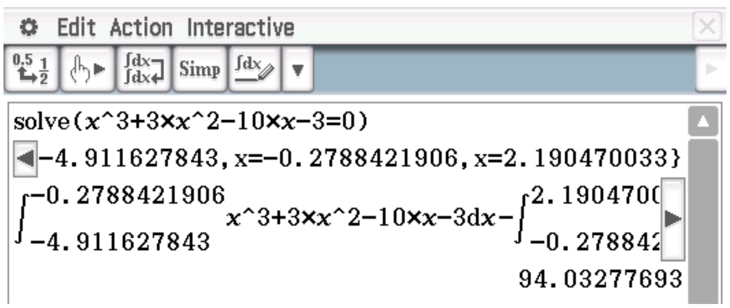
Area between function and the x-axis:

$$-\int_{-1}^{2} 5x^2 - x^3 - 2x - 8\,dx + \int_{2}^{4} 5x^2 - x^3 - 2x - 8\,dx = 21.08\overline{3} \text{ units}^2$$

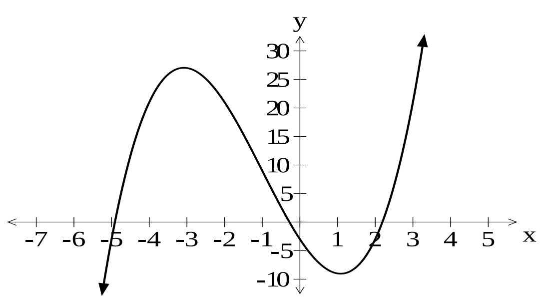
c **TI-Nspire CAS**



ClassPad



$y = x^3 + 3x^2 - 10x - 3$

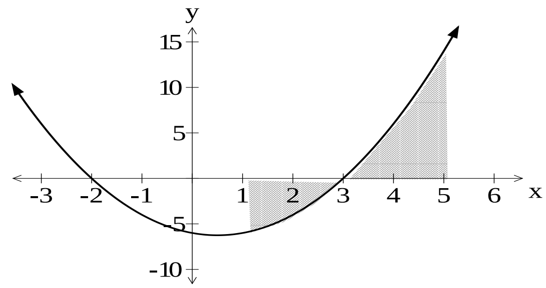


If $y = 0$, $x = ?$, $x = -4.912, -0.279, 2.190$

Area between function and the x-axis:

$$\int_{-4.912}^{-0.279} \left(x^3 + 3x^2 - 10x - 3\right) dx - \int_{-0.279}^{2.190} \left(x^3 + 3x^2 - 10x - 3\right) dx = 94.033 \text{ units}^2$$

21 **a** $y = x^2 - x - 6$, $x = 1$, $x = 5$ and $y = 0$

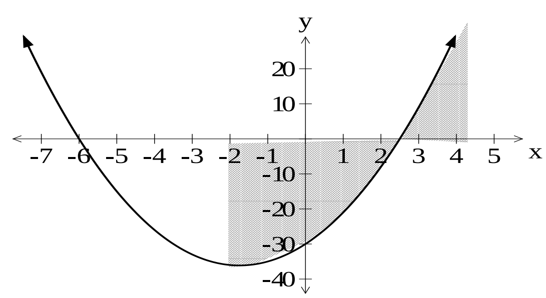


If $y = 0$, $x = ?$, $x = -2, 3$

Required area:

$$-\int_1^3 x^2 - x - 6\,dx + \int_3^5 \left(x^2 - x - 6\right)dx = 20 \text{ units}^2$$

b $y = 2x^2 + 7x - 30$, $x = -2$, $x = 4$ and the x-axis.



If $y = 0$, $x = ?$, $x = -6, 2.5$

Required area:

$$-\int_{-2}^{2.5} 2x^2 + 7x - 30\,dx + \int_{2.5}^{4} 2x^2 + 7x - 30\ dx = 132.75 \text{ units}^2$$

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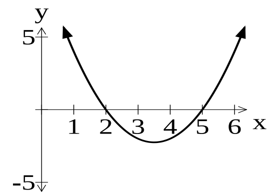
22 **a** $y = x^2 - 7x + 10$

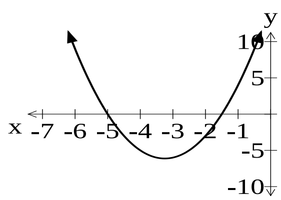


If $y = 0$, $x = ?$, $x = 2, 5$

Area between function and the *x*-axis:

$$-\int_2^5 x^2 - 7x + 10\,dx = 4.5 \text{ units}^2$$

b $y = 2x^2 + 13x + 15$



If $y = 0$, $x = ?$, $x = -5, -1.5$

Area between function and the *x*-axis:

$$-\int_{-5}^{-1.5} \left(2x^2 + 13x + 15\right) dx = 14.29 \text{ units}^2$$

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23 **a** $y = (x + 2)(x - 2)(x - 4)$ by the x-axis

Area between function and the x-axis:

$$\int_{-2}^{2}(x+2)(x-2)(x-4)\,dx - \int_{2}^{4}(x+2)(x-2)(x-4)\,dx$$

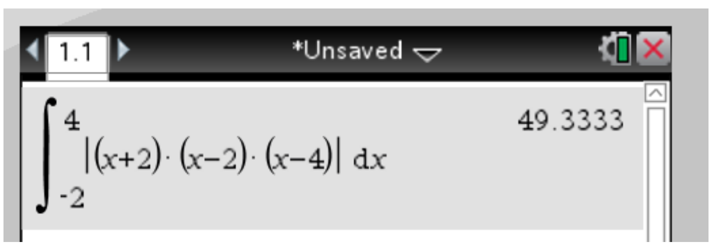
$$= 42\frac{2}{3} - \left(-6\frac{2}{3}\right)$$

$$= 49\frac{1}{3}$$

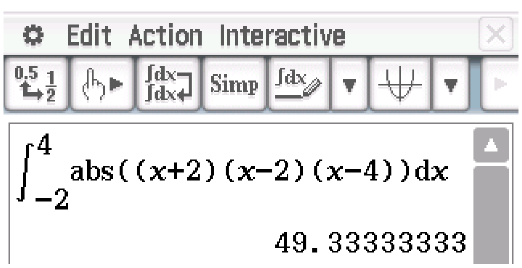
Area between function and the x-axis on a CAS calculator

$$= \int_{-2}^{4}\text{abs}\left[(x+2)(x-2)(x-4)\right]dx$$

TI-Nspire CAS



ClassPad



b $y = (2x + 7)(x + 1)(3 - x)$ by the x-axis

Area between function and the x-axis:

$$-\int_{-3.5}^{-1}(2x+7)(x+1)(x-4)\,dx+\int_{-1}^{3}(2x+7)(x+1)(x-4)\,dx$$
$$=27\frac{11}{32}+96$$
$$=\ 123\frac{11}{32}\ \text{units}^2$$

c $y = (x - 2)(x - 3)^2(x - 4)(x - 6)$ by the x-axis.

Area between function and the x-axis:

$$\int_{2}^{3}(x-2)(x-3)^2(x-4)(x-6)\,dx+\int_{3}^{4}(x-2)(x-3)^2(x-4)(x-6)\,dx$$
$$-\int_{4}^{6}(x-2)(x-3)^2(x-4)(x-6)\,dx$$
$$=\frac{29}{60}+\frac{19}{60}-\left(-17\frac{13}{15}\right)$$
$$=18\frac{2}{3}$$

24 $M(n) = 400(1 - 4e^{-0.015n})$

$$P(n)=\int M(n)\,dn$$
$$P(500)=\int_{0}^{500}400\left(1\ -\ 4e^{-0.015n}\right)dn$$
$$=\$93\,392.33$$

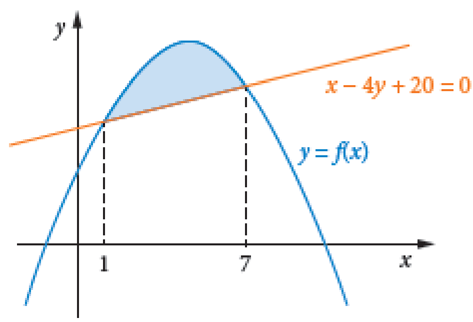
25 $\int_{0}^{0.05}600\,000x\,dx=\left[300\,000x^2\right]_{0}^{0.05}=750\,\text{J}$

Exercise 6.05 Areas between curves

Concepts and techniques

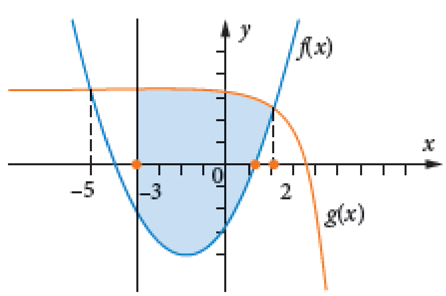
1 C Area = area under the curve $y = f(x)$ – area under the line $y = \frac{1}{4}x + 5$.

$$= \int_1^7 f(x)dx - \int_1^7 \left(\frac{1}{4}x + 5\right)dx$$



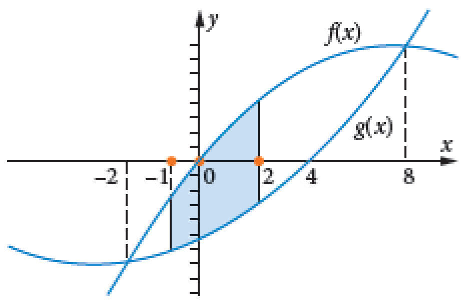
2 B $\int_{-3}^{2} [g(x) - f(x)]dx$

as (area under g) – (area under f) between $x = -3$ and $x = 2$.

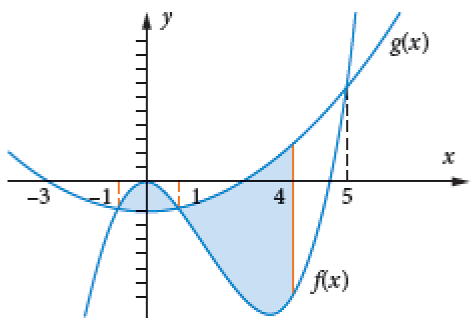


3 D $\int_{-1}^{2} f(x)dx - \int_{-1}^{2} g(x)dx$

as (area under *f*) – (area under *g*) between $x = -1$ and $x = 2$.



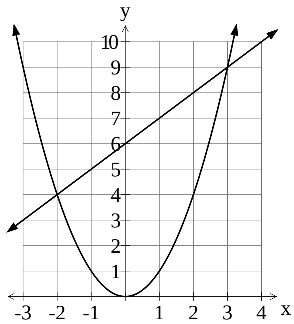
4 A $\int_{-1}^{1} [f(x) - g(x)]dx + \int_{1}^{4} [g(x) - f(x)]dx$



as (area under *f*) – (area under *g*) between –1 and 1 then switched to

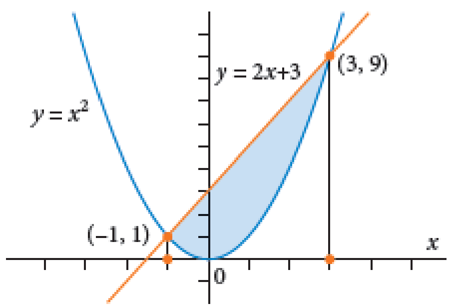
(area under *g*) – (area under *f*) from 1 to 4.

5 The area enclosed between the curve $y = x^2$ and the line $y = x + 6$:



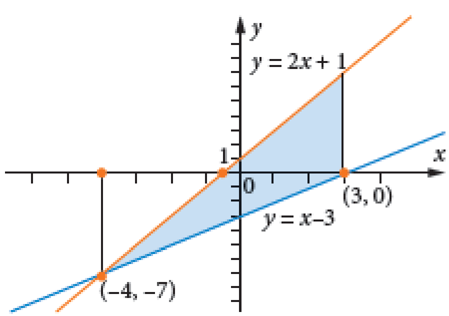
Area = $\int_{-2}^{3}(x+6-x^2)dx = 20.8\overline{3}$ units^2

6 **a**



Area = $\int_{-1}^{3}\left[\left(2x+3\right)-x^2\right]dx = 10.\overline{6}$ units^2

b



Area = $\int_{-4}^{3}\left[\left(2x+1\right)-\left(x-3\right)\right]dx = 24.5$ units^2

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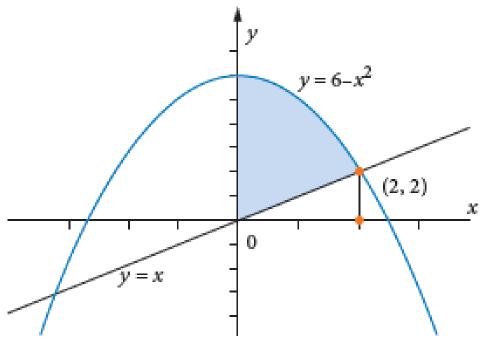
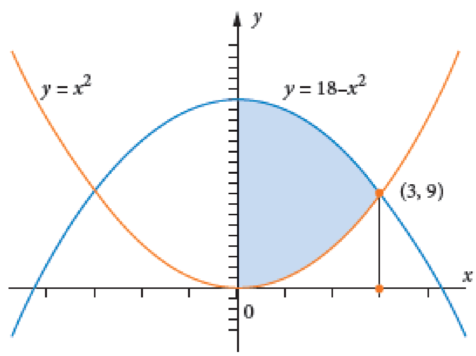
c



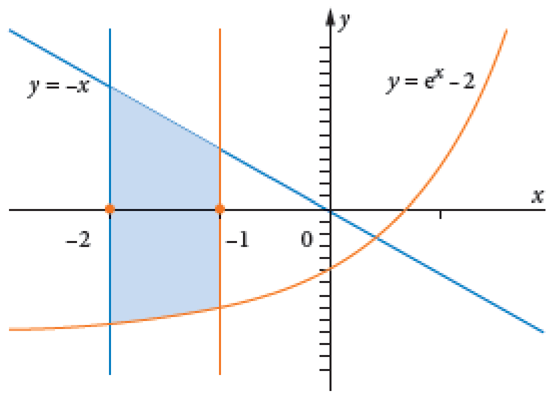
Area = $\int_0^2 \left[\left(-x^2+6\right)-x\right]dx = 7.\overline{3}$ units^2

d



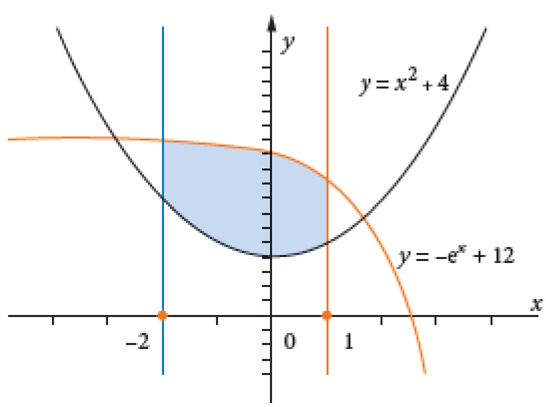
Area = $\int_0^3 \left[\left(18-x^2\right)-x^2\right]dx = 36$ units^2

e



Area = $\int_{-2}^{-1}\left[(-x)-\left(e^{x}-2\right)\right]dx = 3.267$ units^2

f



Area = $\int_{-2}^{1}\left[\left(x^{2}+4\right)-\left(-e^{x}+12\right)\right]dx = 18.417$ units^2

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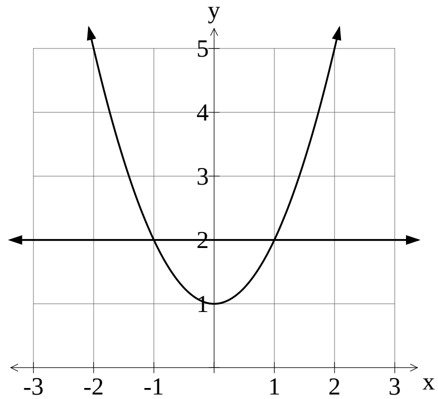
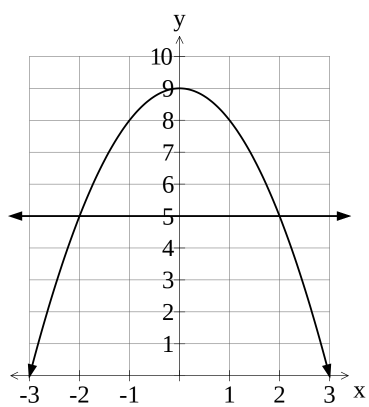
7 $y = 2$ and $y = x^2 + 1$



Area = $\int_{-1}^{1}\left[(2)-\left(x^2+1\right)\right]dx = 1.\overline{3}$ units^2

8 $y = 9 - x^2$ and $y = 5$.



Area = $\int_{-2}^{2}\left[\left(9-x^2\right)-(5)\right]dx = 10.\overline{6}$ units^2

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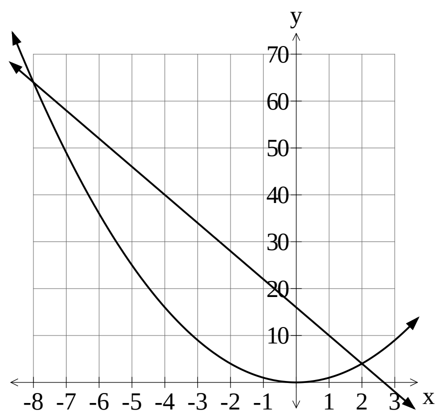
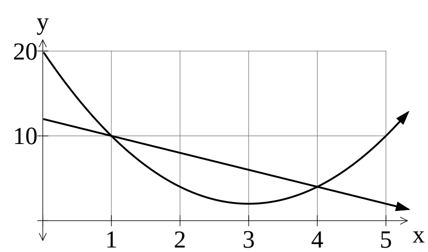
9 $y = x^2$ and $y = -6x + 16$.



Area = $\int_{-8}^{2}\left[\left(-6x+16\right)-\left(x^2\right)\right]dx = 166.\overline{6}$ units^2

10 **a** $y = 2x^2 - 12x + 20$ by $2x + y = 12$



Area = $\int_{1}^{4}\left[\left(-2x+12\right)-\left(2x^2-12x+20\right)\right]dx = 9$ units^2

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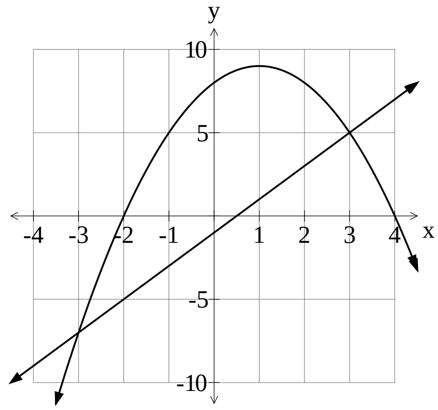
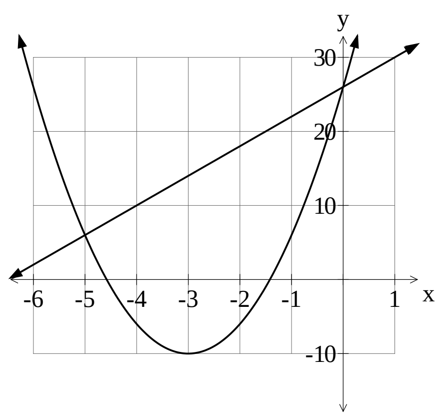
b $f(x) = 2x + 8 - x^2$ by $y = 2x - 1$



Area = $\int_{-3}^{3}\left[\left(2x+8-x^2\right)-\left(2x-1\right)\right]dx = 36$ units^2

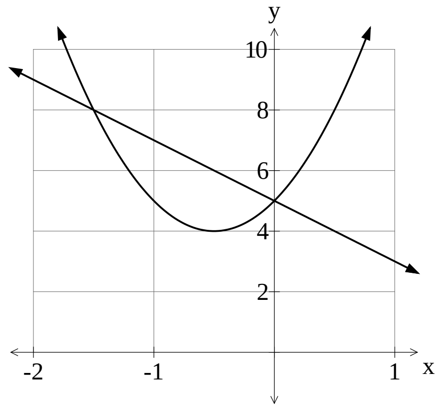
c $f(x) = 4x^2 + 24x + 26$ by $y = 4x + 26$



Area = $\int_{-5}^{0}\left[\left(4x+26\right)-\left(4x^2+24x+26\right)\right]dx = 83.\overline{3}$ units^2

 ISBN 9780170254663

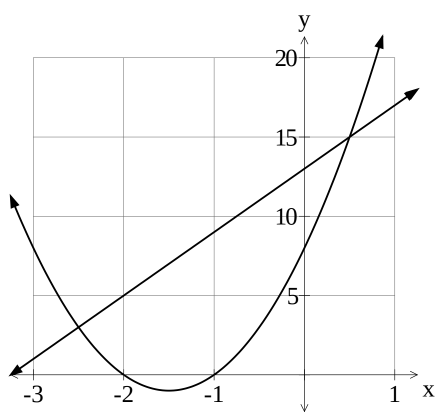
d $f(x) = 4x^2 + 4x + 5$ by $y = 5 - 2x$



Points of intersection (–1.5, 8) and (0, 5).

Area = $\int_{-1.5}^{0}\left[(5-2x)-(4x^2+4x+5)\right]dx = 2.25$ units^2

e $y = 4x^2 + 12x + 8$ by $y = 4x + 13$.



Points of intersection: (–2.5, 3) and (0.5, 15)

Area = $\int_{-2.5}^{0.5}\left[(4x+13)-(4x^2+12x+8)\right]dx = 18$ units^2

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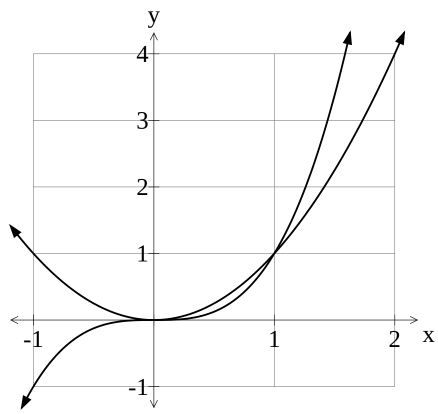
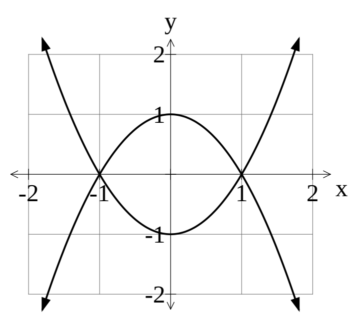
11 $y = x^2$ and $y = x^3$.



Points of intersection: (0, 0) and (1, 11)

Area = $\int_0^1 \left[\left(x^2\right) - \left(x^3\right)\right] dx = 0.08\overline{3}$ units^2

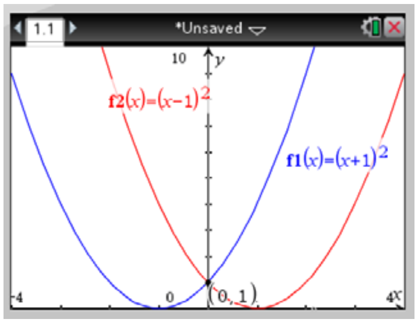
12 $y = 1 - x^2$ and $y = x^2 - 1$.

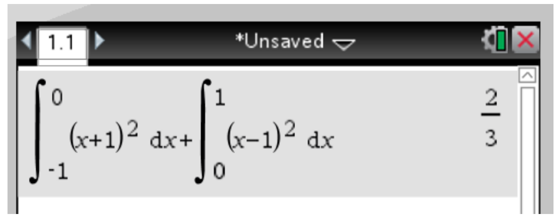


Area = $\int_{-1}^1 \left[\left(1 - x^2\right) - \left(x^2 - 1\right)\right] dx = 2.\overline{6}$ units^2

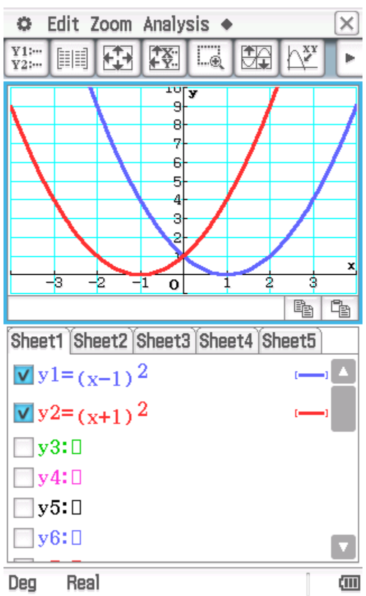
 ISBN 9780170254663

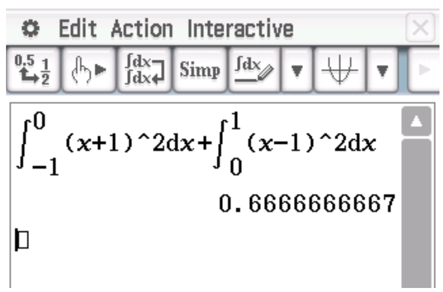
13 TI-Nspire CAS



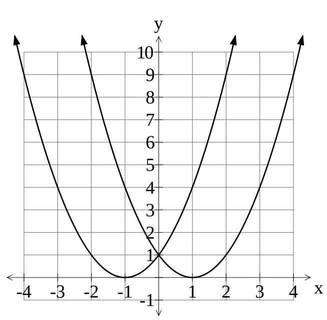


ClassPad



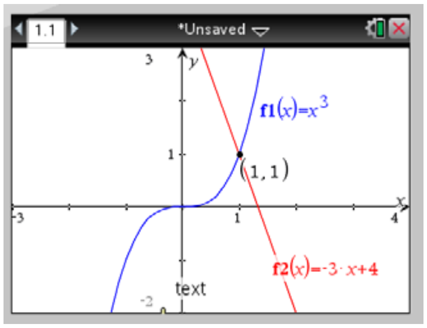


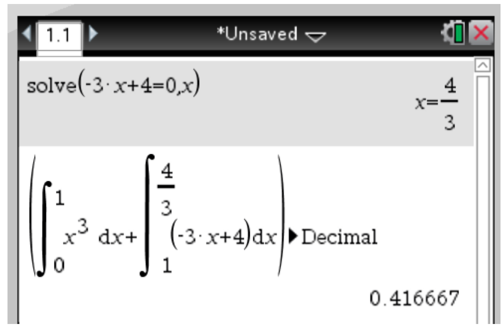
$y = (x-1)^2$ and $y = (x+1)^2$.



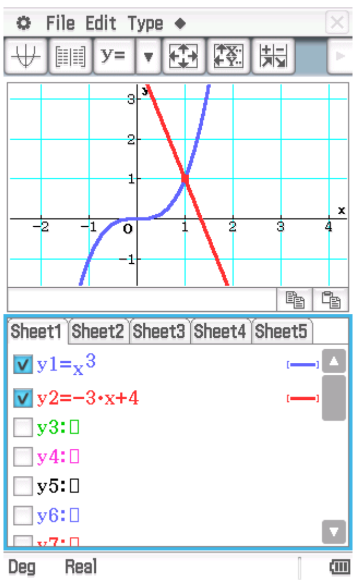
$$\text{Area} = \int_{-1}^{0}(x+1)^2\,dx + \int_{0}^{1}(x-1)^2\,dx = \left[\frac{x^3}{3}+x^2+x\right]_{-1}^{0} + \left[\frac{x^3}{3}-x^2+x\right]_{-1}^{0} = \frac{1}{3}+\frac{1}{3} = \frac{2}{3}$$

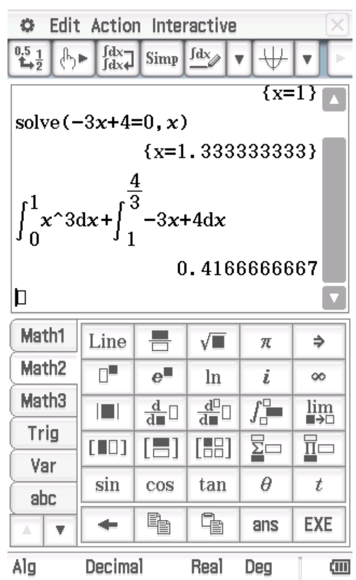
14 **TI-Nspire CAS**



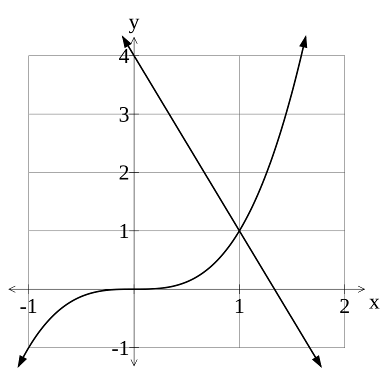


ClassPad





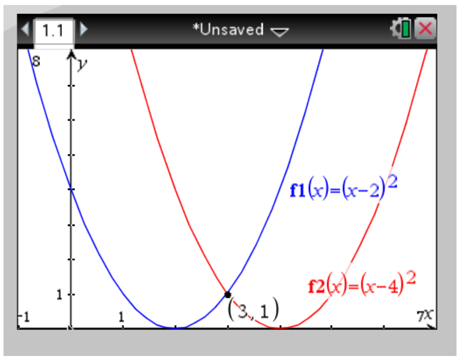
$y = x^3$, the x-axis and the line $y = -3x + 4$.

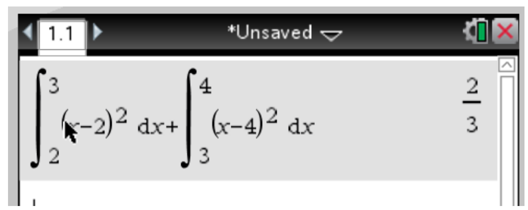


Point of intersection: (1, 11). The x-intercept of the line is $\left(\frac{4}{3}, 0\right)$.

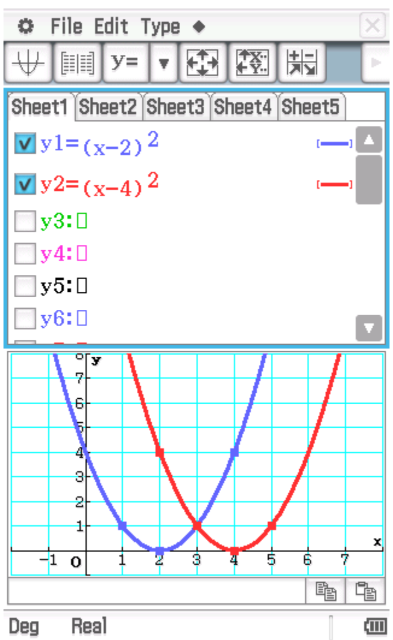
Area = $\int_0^1 x^3 dx + \int_1^{\frac{4}{3}} -3x + 4\, dx = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ units^2

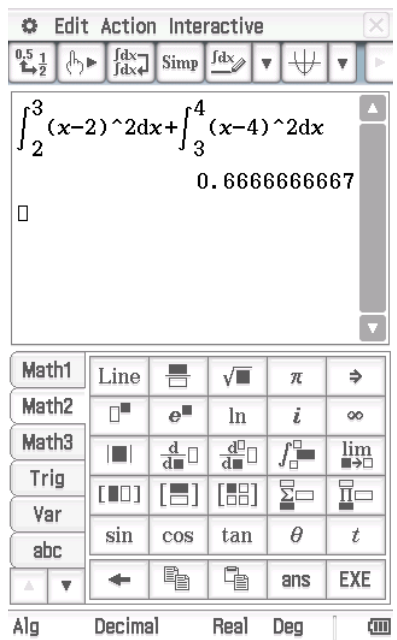
15 TI-Nspire CAS



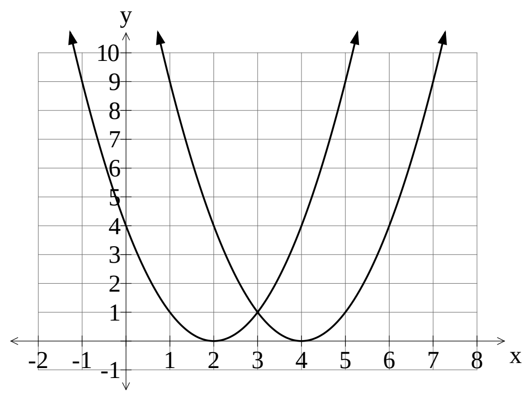


ClassPad



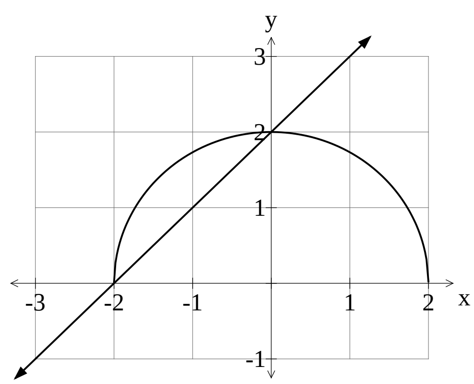


$y = (x - 2)^2$ and $y = (x - 4)^2$.



Area = $\int_2^3 (x-2)^2 dx + \int_3^4 (x-4)^2\, dx = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

16 $y = \sqrt{4 - x^2}$ and the line $x - y + 2 = 0$.



Area = $\int_{-2}^{0} \left[\sqrt{4 - x^2} - \left(x + 2 \right) \right] dx = 1.142$ units^2

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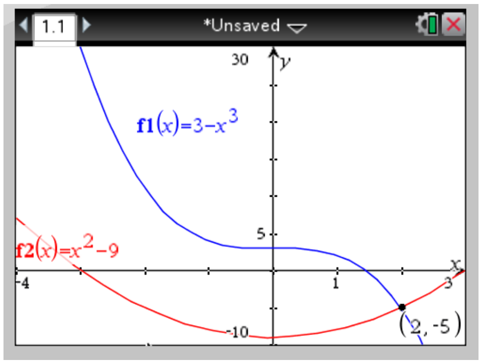
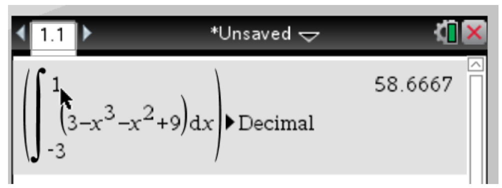
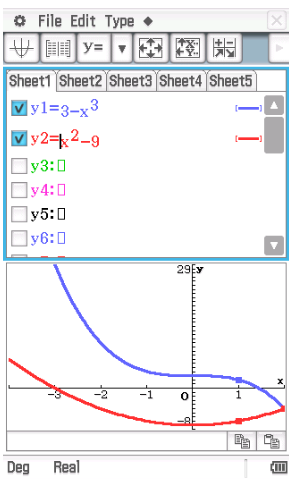
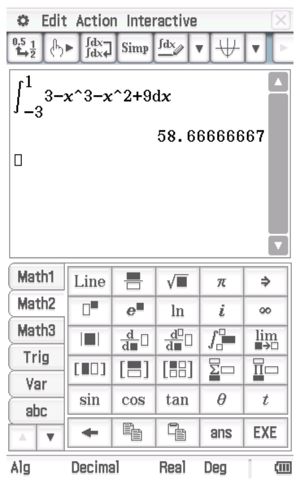
17 TI-Nspire CAS



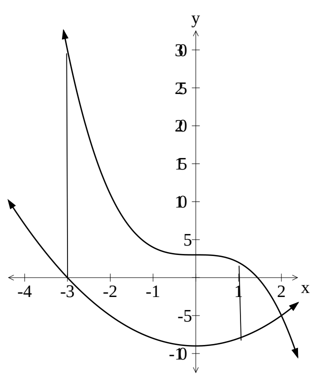


ClassPad





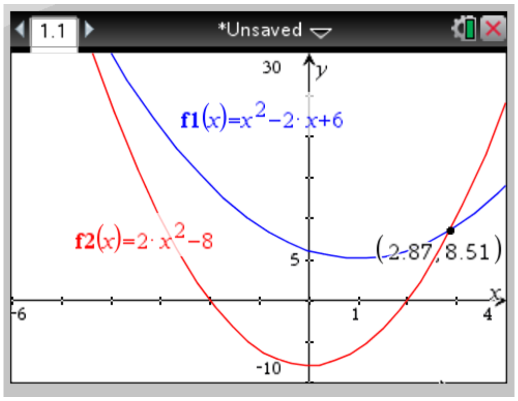
a $f(x) = 3 - x^3$, $g(x) = x^2 - 9$, $x = -3$ and $x = 1$

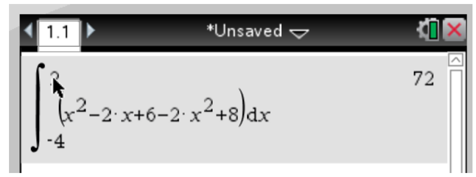


$$\text{Area} = \int_{-3}^{1}\left[\left(3 - x^3\right) - \left(x^2 - 9\right)\right]dx = 58.\overline{6}\ \text{units}^2$$

ISBN 9780170254663

b TI-Nspire CAS





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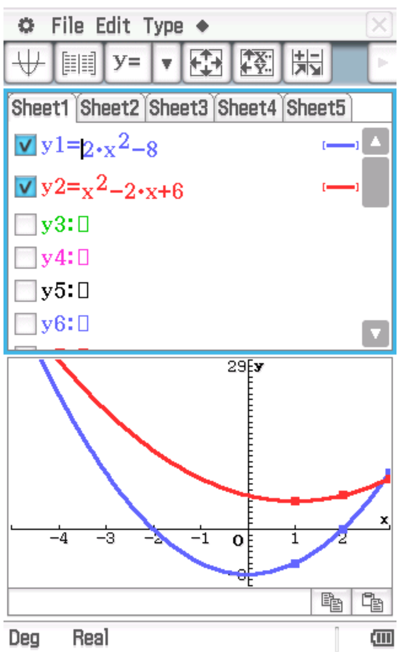

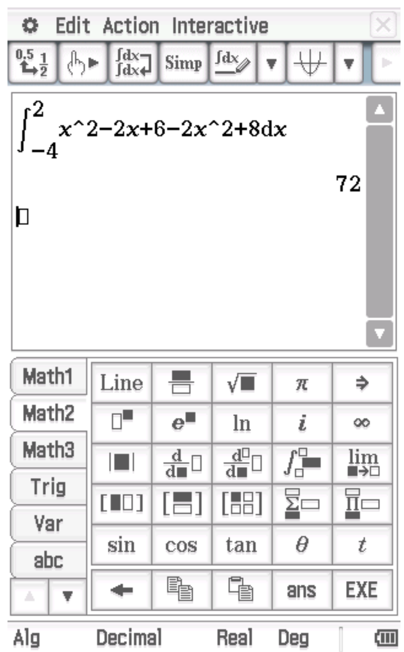

$f(x) = 2x^2 – 8$, $g(x) = x^2 – 2x + 6$, $x = –4$ and $x = 2$

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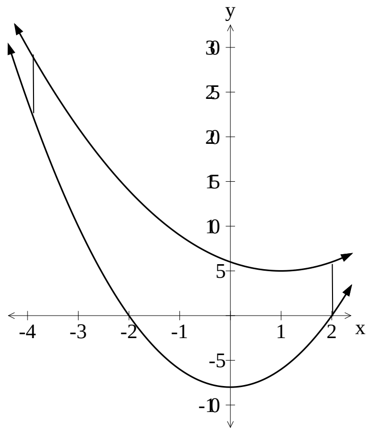
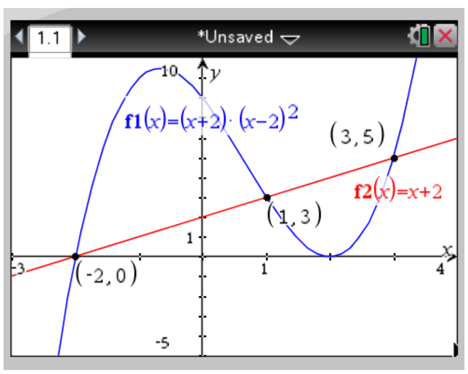
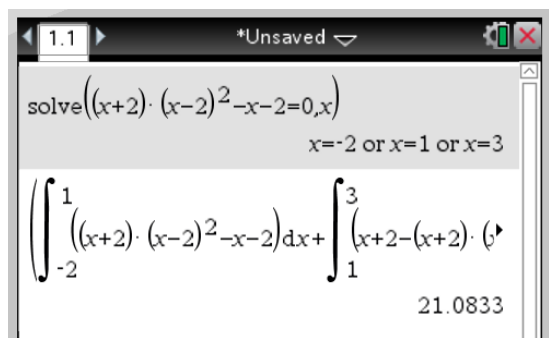


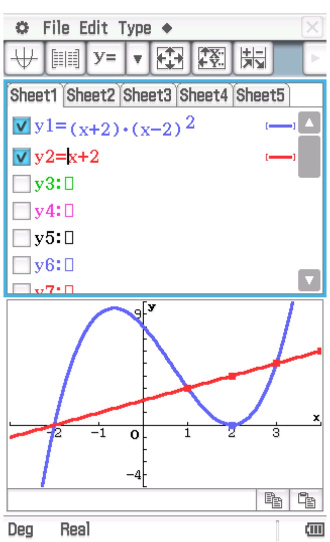
$$\text{Area} = \int_{-4}^{2}\left[\left(x^2-2x+6\right)-\left(2x^2-8\right)\right]dx = 72 \text{ units}^2$$

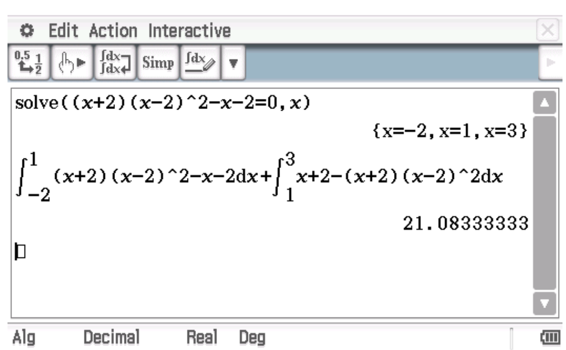
c TI-Nspire CAS



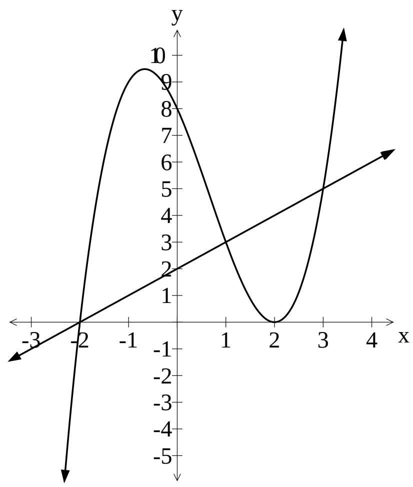


ClassPad





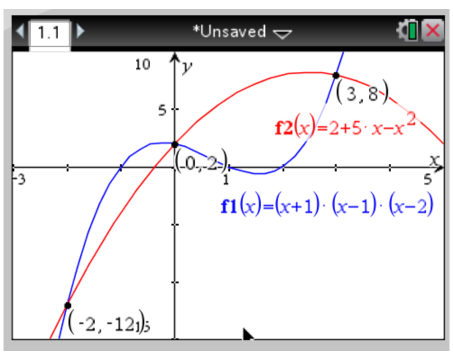
$f(x) = (x+2)(x-2)^2$ and $y = x + 2$

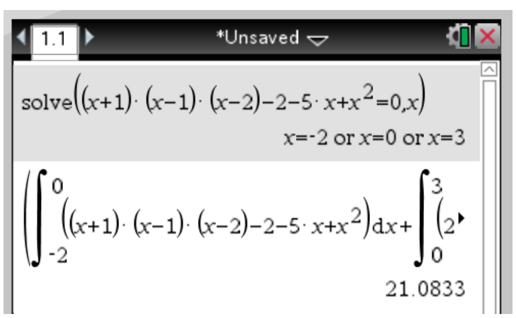


Points of intersection: (–2, 0), (1, 3) and (3, 5).

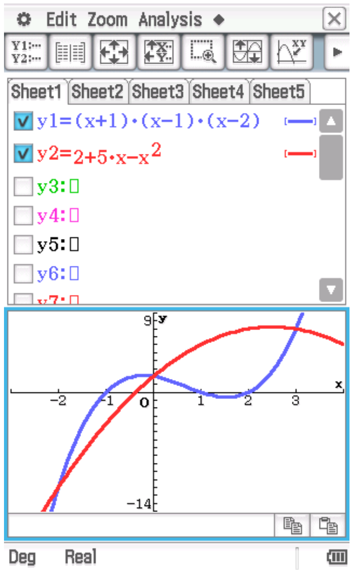
Area = $\int_{-2}^{1}\left[(x+2)(x-2)^2-(x+2)\right]dx+\int_{1}^{3}\left[x+2-(x+2)(x-2)^2\right]dx = 21\tfrac{1}{12}$ units^2

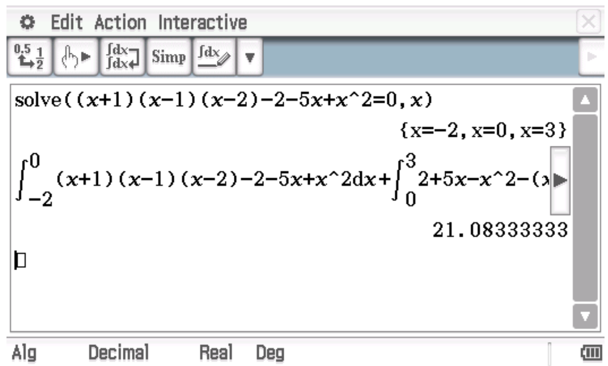
d TI-Nspire CAS



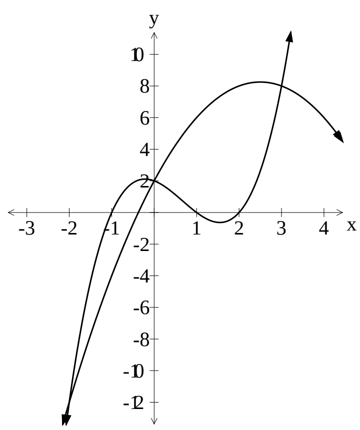


ClassPad





$y = (x + 1)(x - 1)(x - 2)$ and $y = 2 + 5x - x^2$



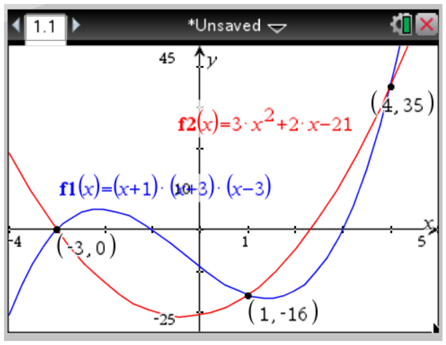
Points of intersection: (–2, –12), (0, 2) and (3, 8).

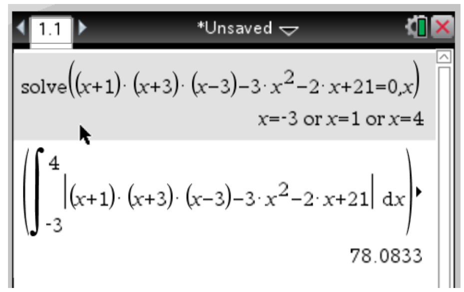
$$\text{Area} = \int_{-2}^{0}\left[(x+1)(x-1)(x-2)-\left(2+5x-x^2\right)\right]dx$$

$$+\int_{0}^{3}\left[\left(2+5x-x^2\right)-(x+1)(x-1)(x-2)\right]dx$$

$$= 21.08\overline{3}\ \text{units}^2$$

e





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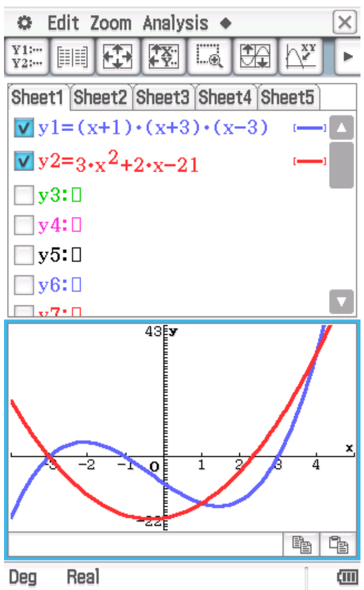
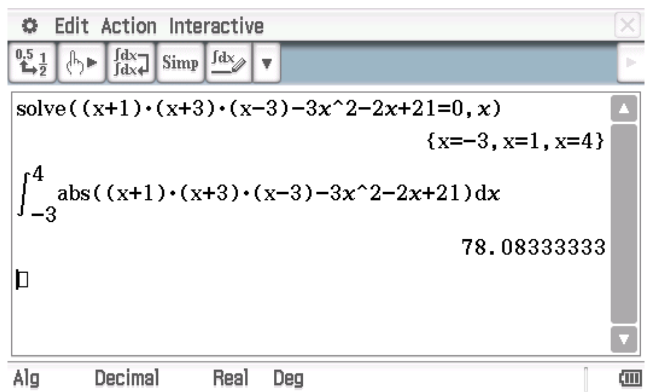
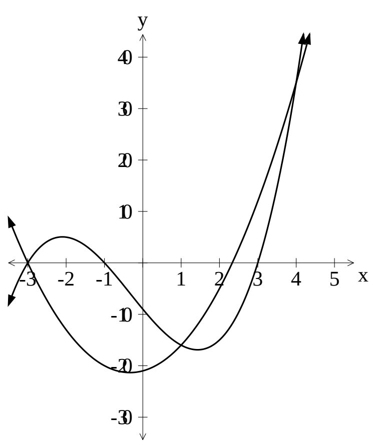




$y = 3x^2 + 2x - 21$ and $y = (x + 1)(x + 3)(x - 3)$

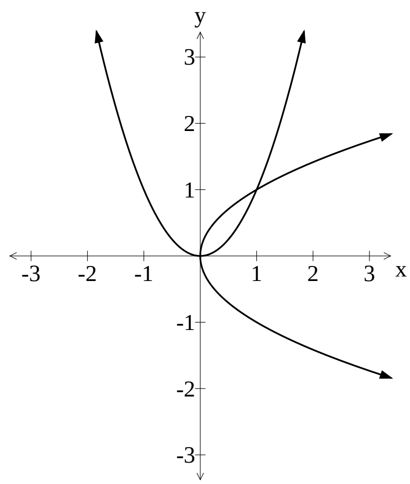


Points of intersection: (–3, 0), (1, –16) and (4, 35).

Area = $\int_{-3}^{4}\left|(x+1)(x+3)(x-3)-\left(2x^2+2x-21\right)\right|dx = 78.08\overline{3}$ units^2

Reasoning and communication

18 $y = x^2$ and $x = y^2$.



Area = $\int_{0}^{1}\sqrt{x}-x^2dx = \frac{1}{3}$ units^2

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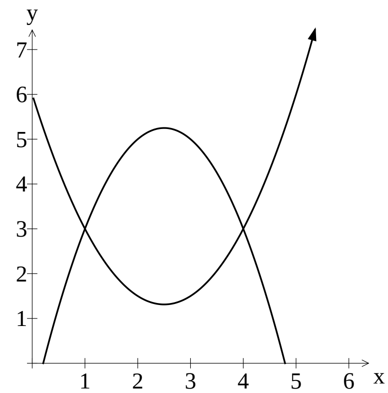
19 $y = 0.75x^2 - 3.75x + 6$ and $y = 5x - 1 - x^2$



Points of intersection (1, 3) and (4, 3).

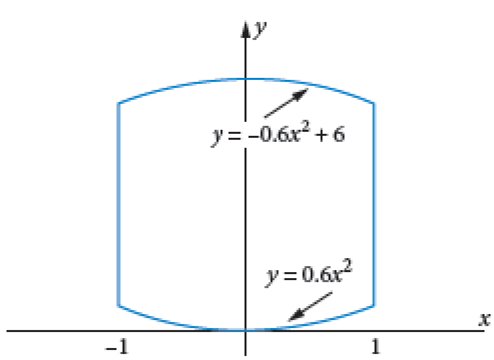
Area = $\int_1^4 (5x-1-x^2)-(0.75x^2-3.75x+6)\,dx = 7.875\ \text{cm}^2$

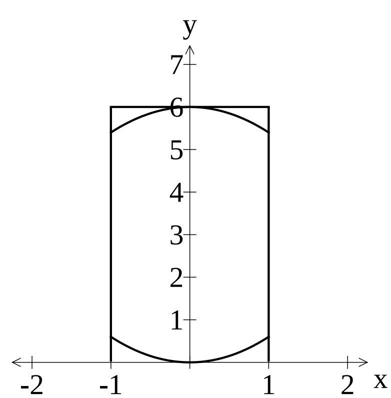
Volume = 7.875 × 32 = 252 cm^3

Density is 7920 kg/m^3

Mass = $\dfrac{252}{1000\,000}\times 7920\ \text{kg} = 1.99584\ \text{kg}$

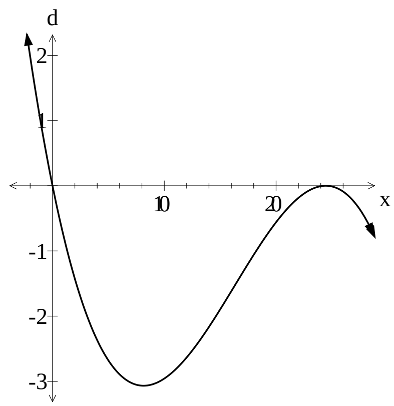
20





Area discarded = $2 \times \left[\int_{-1}^{1} (0.6x^2+6)\,dx\right] = 0.8\,\text{m}^2$

21 $d = -0.007x(0.45x - 11)^2$

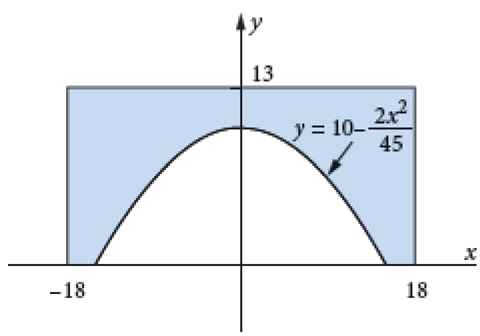


x-intercepts 0, $24\frac{4}{9}$

Volume = $-\left[\int_0^{24\frac{4}{9}} -0.007x\left(0.45x-11\right)^2 dx\right] \times 0.3 \times (60 \times 30)$ m^3 = 22 774.88… m^3

1 m^3 = 1000 kL, so volume in 30 minutes is about 22.775×10^6 L = 22.8 ML

22



a Area of the cross section

$$= 2\times\left\{13\times18-\int_0^{15}10-\frac{2x^2}{45}dx\right\}=2\left\{234-100\right\}=268 \text{ m}^2$$

b Volume = 268 × 25 = 6700 m^3

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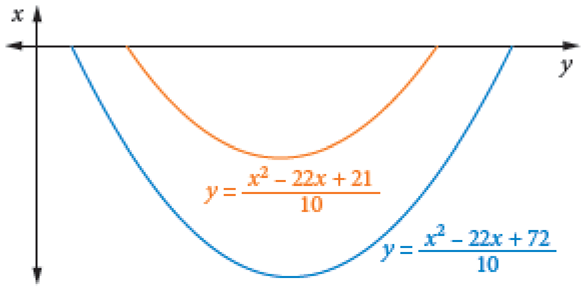
23 $y = \dfrac{x^2 - 22x + 21}{10}$ and $y = \dfrac{x^2 - 22x + 72}{10}$



$$\text{Area} = \frac{1}{10}\left| \int_1^{21}\left(x^2 - 22x + 21\right)dx - \int_4^{18}\left(x^2 - 22x + 72\right)dx \right| = \frac{1}{10}\left| -1333.\bar{3} - \left(-457.\bar{3}\right)\right| = 87.6$$

Cost = 87.6 × 0.15 × \$350 = \$4599

Exercise 6.06 Total change

Concepts and techniques

1 B The total change in the volume of oil in the tank

$$= \int_0^5 1000e^{-0.1t}\,dt$$

2 D $P'(t) = 6 + \sqrt{10t}$

$$P(t) = \int_0^{10} 6 + \sqrt{10t}\,dt$$

$$= \left[6t + \frac{2\sqrt{10}}{3} t^{\frac{3}{2}} \right]_0^{10}$$

3 $\int_0^{10} H'(t)dt$ represents the change in height in cm of the fertilizer in 10 hours.

4 $\int_2^8 B'(t)dt$ represents the increase in bacteria from $t = 2$ to $t = 8$ hours.

5 $150 + \int_0^4 P'(t)dt$

6 **a** $\int_0^5 3500e^{-0.4t}dt = 3500\left[\frac{e^{-0.4t}}{-0.4}\right]_0^5 = \frac{3500}{-0.4}\left[e^{-0.4t}\right]_0^5 = -8750\times(-0.865) = 7566\text{ L}$

b $\int_5^{10} 3500e^{-0.4t}dt = \frac{3500}{-0.4}\left[e^{-0.4t}\right]_5^{10} = -8750\times(-0.1170) = 1024\text{ L}$

c $e^{-0.4t}$ is a decreasing function.

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7 **a** $\int_0^{10} 4t+1\,dt = \left[2t^2+t\right]_0^{10} = 200+10-0 = 210$

i.e. 21 000 rabbits

b $21 = \int_0^{t} 4t+1\,dt = \left[2t^2+t\right]_0^{t} = 2t^2+t$

$21 = 2t^2+t \quad \Rightarrow \quad (2t+7)(t-3) = 0$

t = 3 months

8 $C'(x) = 25 - \frac{1}{2}x$

$$C(50) = \int_0^{50} 25 - \frac{x}{2}\,dx = \left[25x - \frac{x^2}{4}\right]_0^{50} = 625$$

Cost for 50 components is $625 000.

9 $R'(x) = 12 - 3x^2 + 4x$

$$R(x) = \int_0^{x} 12 - 3x^2 + 4x\,dx = \left[12x - x^3 + 2x^2\right]_0^{x}$$
$$= 12x - x^3 + 2x^2$$
$$R(4) = 16$$

Therefore, the total revenue from the sale of the first 400 units is $16 000.

Reasoning and communication

10 $W'(t) = \frac{1}{75}(20t - t^2 + 600)$ so $W(t) = \frac{1}{75}\left(10t^2 - \frac{t^3}{3} + 600t\right) + c$ L

$W(0) = 200$, so $W(t) = \frac{1}{75}\left(10t^2 - \frac{t^3}{3} + 600t\right) + 200$ L

After 24 hours, $W(24) = 407.36$ L

11 $R(x) = \int 10 - 0.002x \, dx = 10x - 0.001x^2 + c$

$R(0) = 0$

$\therefore \quad R(x) = 10x - 0.001x^2$

$C(x) = 2x + k$

But $k = 7000$ so $C(x) = 2x + 7000$

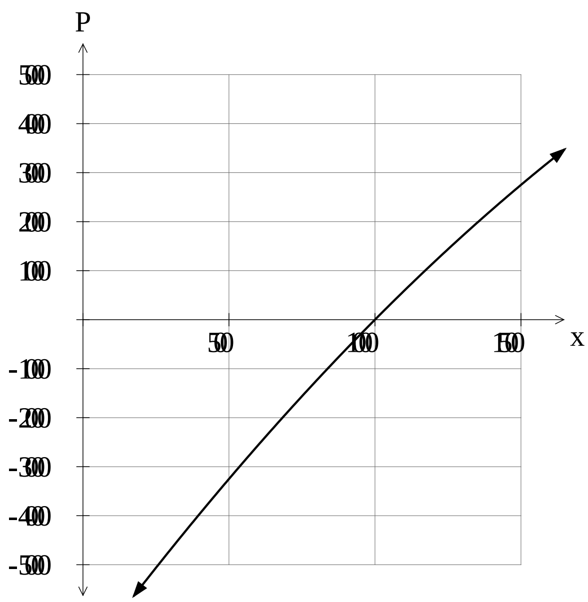
$P(x) = R(x) - C(x)$

$P(x) = 10x - 0.001x^2 - (7000 + 2x)$

$P(x) = 8x - 0.001x^2 - 7000$

$P(1000) = 0$

The total profit for the first 1000 toy cars produced is \$0, i.e. the break-even point.



12 $C'(x) = 5 + 16x - 3x^2$

$C(x) = \int 5 + 16x - 3x^2 \, dx = 5x + 8x^2 - x^3 + c$

$C(5) = 500 \quad \Rightarrow c = ?$

$500 = 100 + c$

$C(x) = 5x + 8x^2 - x^3 + 400$

13 $C'(x) = kx + 5000$

$$C(x) = \int kx\ dx = \frac{kx^2}{2} + 5000$$

$$C(24) = 5144 \ \Rightarrow c = ?$$

$$5144 = \frac{k(24)^2}{2} + 5000$$

$$k = 0.5$$

$$C(x) = \frac{x^2}{4} + 5000$$

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Exercise 6.07 Application of integration to motion

Concepts and techniques

1 $\int_{2}^{10} v(t)\,dt$ represents the displacement of the particle from $t = 2$ to $t = 10$ seconds.

2 At $t = 0$, $v = 0$

$a = 6$ m/s^2

$v = \int 6dt = 6t + c$

At $t = 0$

$0 = 0 + c$

$v = 6t$

$x = \int 6t\,dt = 3t^2 + c$

Displacement at $t = 4.1$, $x = 3(4.1)^2 = 50.43$ m

Note: Displacement means $c = 0$

3 $v = 3t^2 + 2t + 1$

At $t = 0$, $x = -2$

$x = \int 3t^2 + 2t + 1dt = t^3 + t^2 + t + c$

At $t = 0$

$-2 = 0 + c$

$x = t^3 + t^2 + t - 2$

$x_5 = 125 + 25 + 5 - 2$

$= 153$ m

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4 $a = -9 \sin(3t)$ cm/s^2.

At $t = 0$, $v = 5$ cm/s, $x = -3$cm

$$v = \int -9\sin(3t)\,dt = 3\cos(3t) + c$$
$$\text{At } t = 0$$
$$5 = 3 + c$$
$$v = 3\cos(3t) + 2$$
$$x = \int 3\cos(3t)\,dt = \sin(3t) + c$$
$$\text{At } t = 0$$
$$-3 = \sin(0) + c$$
$$x = \sin(3t) - 3$$
$$x_\pi = \sin(3\pi) - 3$$
$$= -3$$

It is 3 cm to the left of the origin.

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5 $v = 4\cos(2t)$ m/s.

At $t = \pi$ s, $x = 3$

a $x = \int 4\cos(2t)dt = 2\sin(2t) + c$

At $t = \pi$

$x = 2\sin(2t) + c$

$3 = 0 + c$

$x = 2\sin(2t) + 3$

$x_{\frac{\pi}{6}} = 2\sin\left(\frac{\pi}{3}\right) + 3 = 2\left(\frac{\sqrt{3}}{2}\right) + 3$

$x_{\frac{\pi}{6}} = \sqrt{3} + 3$

b $v = 4\cos(2t)$ m/s.

$a = -8\sin(2t)$

$a_{\frac{\pi}{6}} = -8\sin\left(\frac{\pi}{3}\right) = -8\left(\frac{\sqrt{3}}{2}\right)$

$= -4\sqrt{3}$ m/s^2

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6 At $t = 0$, $v = 20$ m/s, $x = 300$

a $a = -9.8\,\text{m/s}^2$

$v = -9.8t + c$

At $t = 0$

$20 = c$

$v = -9.8t + 20$

$x = -4.9t^2 + 20t + c$

At $t = 0$

$300 = c$

$x = -4.9t^2 + 20t + 300$

$x_5 = 277.5$ m

b $v = -9.8t + 20$

$v_5 = -9.8t + 20 = -29\,\text{m/s}$

c Greatest height at $v = 0$

$t = \dfrac{20}{9.8} = 2.04$ s

7 $v_0 = 14\,\text{m/s},\ x_0 = 0$

a $a = -9.8\ \text{m/s}^2$

$v = -9.8t + c$

At $t = 0$

$14 = c$

$v = -9.8t + 14$

$x = -4.9t^2 + 14t + c$

At $t = 0$

$0 = c$

$x = -4.9t^2 + 14t$

At $v = 0$,

$t = \dfrac{14}{9.8} = 1.429$

height = ?

$x = -4.9(1.429)^2 + 14(1.429)$

$= 10\ \text{m}$

b At $x = 0$, $t = ?$

$0 = -4.9t^2 + 14t = t(-4.9t + 14)$

$t > 0, t = \dfrac{14}{4.9} = 2.857$

c $v_{2.857} = -9.8(2.857) + 14 = -14\,\text{m/s}$

8 At $t = 0$, $v = 0$, $a = –9.8$

At $t = 2.5$ s, $x = 0$

$$v = -9.8t + c$$
$$\text{At } t = 0$$
$$0 = c$$
$$v = -9.8t$$
$$x = -4.9t^2 + c$$
$$\text{At } t = 2.5, x = 0$$
$$0 = -4.9(2.5)^2 + c$$
$$c = 30.625$$
$$x = -4.9t^2 + 30.625$$
$$\text{At } t = 0$$
$$x = 30.63 \text{ m} \;\; (2 \text{ dp})$$

9 At $t = 0$, $v = 2.1 \times 10^3$ m/s, $a = –9.8$, $x = 0$

$$v = -9.8t + c$$
$$\text{At } t = 0$$
$$2.1 \times 10^3 = c$$
$$v = -9.8t + 2.1 \times 10^3$$
$$x = -4.9t^2 + 2.1 \times 10^3 t + c$$
$$\text{At } t = 0, x = 0$$
$$c = 0$$
$$x = -4.9t^2 + 2.1 \times 10^3 t$$
Maximum height when $v = 0$
$$0 = -9.8t + 2.1 \times 10^3$$
$$t = 214.287 \text{ secs}$$
$$x_{214.287} = 224\,999.991$$
$$= 225\,000 \text{ m}$$

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10 $a = -e^{2t}$ cm/s^2

$v_0 = 0$, $x_0 = 0$

$$v = \frac{-e^{2t}}{2} + c$$

At $t = 0, v = 0$

$$0 = -\frac{1}{2} + c$$

$$v = \frac{-e^{2t}}{2} + \frac{1}{2}$$

$$x = \frac{-e^{2t}}{4} + \frac{x}{2} + c$$

At $t = 0$

$$0 = -\frac{1}{4} + 0 + c$$

$$x = -\frac{e^{2t}}{4} + \frac{x}{2} + \frac{1}{4}$$

$$x_4 = -\frac{e^{8}}{4} + \frac{4}{2} + \frac{1}{4}$$

$$= -\frac{e^{8}}{4} + 2\frac{1}{4}$$

$$= -742.989$$

$$\approx -743 \text{ cm}$$

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11 $a = e^{3t}$.

$v_0 = -2$ m/s, $x_0 = 0$

$$v = \frac{e^{3t}}{3} + c$$

At $t = 0, v = -2$

$$-2 = \frac{1}{3} + c$$

$$v = \frac{e^{3t}}{3} - 2\frac{1}{3}$$

$$x = \frac{e^{3t}}{9} - 2\frac{1}{3}x + c$$

At $t = 0$

$$0 = \frac{1}{9} + 0 + c$$

$$x = \frac{e^{3t}}{9} - 2\frac{1}{3}x + \frac{1}{9}$$

$$x_3 = \frac{e^9}{9} - 7 + \frac{1}{9}$$

$$= 893 \text{ (3 sig figs)}$$

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12 $a = 25e^{5t}$ m/s^2

a $v_0 = 5$ m/s, $x_0 = 1$

$$v = 5e^{5t} + c$$
$$\text{At } t = 0, v = 5$$
$$5 = 5 \times 1 + c \quad \Rightarrow c = 0$$
$$v = 5e^{5t}$$
$$v_9 = 5e^{45} \text{m/s}$$

b $x_6 = ?$

$$x = e^{5t} + c$$
$$\text{At } t = 0$$
$$1 = 1 + c$$
$$x = e^{5t}$$
$$x_6 = e^{30} \text{ m}$$

Reasoning and communication

13 $v(t) = \dfrac{g}{2}(1 - e^{-2t})$

$$x(t) = -4.9\left(t + \frac{e^{-2t}}{2}\right) + c$$

Finding displacement $\Rightarrow c = 0$

$$x(100) = -4.9\left(100 + \frac{e^{-200}}{2}\right) = -490\,\text{m}$$

The manila folder falls 490 m in the first 100 s.

14 **a** $a = -9.8$, max $x = 2$ m, initial velocity = v_0

$v = -9.8t + c$

At $t = 0, v = v_0$

$v = -9.8t + v_0$

$x = -4.9t^2 + v_0 t + c$

At $t = 0, x = 0$

$c = 0$

$x = -4.9t^2 + v_0 t$

Maximum height when $v = 0$

$0 = -9.8t + v_0$

$t = \frac{v_0}{9.8}$ s

$2 = -4.9\left(\frac{v_0}{9.8}\right)^2 + v_0 \frac{v_0}{9.8}$

$v_0 > 0, v_0 = 6.26$ m/s

b $a = -1.6$, max $x = ?$ m, initial velocity = v_0

$v = -1.6t + c$

At $t = 0, v = v_0$ Assume $v_0 = 6.26$ m/s

$v = -1.6t + v_0$

$x = -0.8t^2 + v_0 t + c$

At $t = 0, x = 0$

$c = 0$

$x = -0.8t^2 + v_0 t$

Maximum height when $v = 0$

$0 = -1.6t + v_0$

$t = \frac{6.26}{1.6} = 3.91$ s

$x = -0.8(3.91)^2 + 6.26 \times 3.91$

Max x is 12.25 m.

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15 $v = t^2(t^3 + 1) = t^5 + t^2$ cm/s

$x_0 = 2$cm

a $a = 5t^4 + 2t$

$a_1 = 7\text{ cm/s}^2$

b $x = \int t^2\left(t^3 + 1\right)dt$

$$x = \int t^5 + t^2 dt$$

$$x = \frac{t^6}{6} + \frac{t^3}{3} + c$$

At $t = 0$

$2 = 0 + c$

$$x = \frac{t^6}{6} + \frac{t^3}{3} + 2$$

$$x_2 = \frac{64}{6} + \frac{8}{3} + 2$$

$$= 15.\overline{3}$$

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16 $a = \cos^2\left(t+\frac{\pi}{4}\right) - \sin^2\left(t+\frac{\pi}{4}\right)$

At $t = 0$, $v = 0$, $x = 0$

a $v = \int \cos^2\left(t+\frac{\pi}{4}\right) - \sin^2\left(t+\frac{\pi}{4}\right) dt$

$$v = \int \cos\left(2t+\frac{\pi}{2}\right) dt$$

$$v = \frac{\sin\left(2t+\frac{\pi}{2}\right)}{2} + c$$

At $t = 0$

$$0 = \frac{\sin\left(\frac{\pi}{2}\right)}{2} + c \quad \Rightarrow c = -\frac{1}{2}$$

$$v = \frac{\sin\left(2t+\frac{\pi}{2}\right)}{2} - \frac{1}{2}$$

At $t = \frac{\pi}{2}$

$$v = \frac{1}{2} \times \sin\left(\pi+\frac{\pi}{2}\right) - \frac{1}{2}$$

$$= -\frac{1}{2} - \frac{1}{2}$$

$$= -1 \text{ cm/s}$$

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b $v=\frac{\sin\left(2t+\frac{\pi}{2}\right)}{2}-\frac{1}{2}$

$$x=-\frac{\cos\left(2t+\frac{\pi}{2}\right)}{4}-\frac{t}{2}+c$$

At $t=0$

$$0=-\frac{\cos\left(\frac{\pi}{2}\right)}{4}-0+c\Rightarrow c=0$$

$$x=-\frac{\cos\left(2t+\frac{\pi}{2}\right)}{4}-\frac{t}{2}$$

$$x_{\frac{\pi}{2}}=-\frac{\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}{4}-\frac{\frac{\pi}{4}}{2}$$

$$=\frac{1}{4}-\frac{\pi}{8}$$

c $v=-\frac{1}{2}$ cm/s, t = ?

$$-\frac{1}{2}=\frac{\sin\left(2t+\frac{\pi}{2}\right)}{2}-\frac{1}{2}$$

$$\therefore\ \sin\left(2t+\frac{\pi}{2}\right)=0$$

$$2t+\frac{\pi}{2}=0,\pi,2\pi,3\pi,\ldots\ \ t>0$$

$$2t=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\ldots$$

$$t=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\ldots$$

$t=\frac{2n+1}{4}$ where n is a counting number

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Chapter 6 Review

Multiple choice

1 E $5x^3 - 3x + 4$ $f(x) = \int 15x^2 - 3dx = 5x^3 - 3x + c$

$(1, 6) \Rightarrow 6 = 5 - 3 + c$

$c = 4$

$f(x) = 15x^2 - 3x + 4$

2 E $4.8x^2\sqrt{x} + c$ $\int 12x\sqrt{x}\,dx = 12\int x^{\frac{3}{2}}dx = 12x^{\frac{5}{2}} \times \frac{2}{5} + c = 4.8x^2\sqrt{x} + c$

3 A $\int (3x^3 - 5x + 2)dx = \int 3x^3 dx - \int 5x\,dx + \int 2dx$ as integration is distributive.

4 C $\frac{dy}{dx} = \frac{mx^3}{2} + 3x$

$y = \int \frac{mx^3}{2} + 3x\,dx$

$y = \frac{mx^4}{8} + 3x^2 + c$

$(0, 4) \Rightarrow 4 = c$

$y = \frac{mx^4}{8} + 3x^2 + 4$

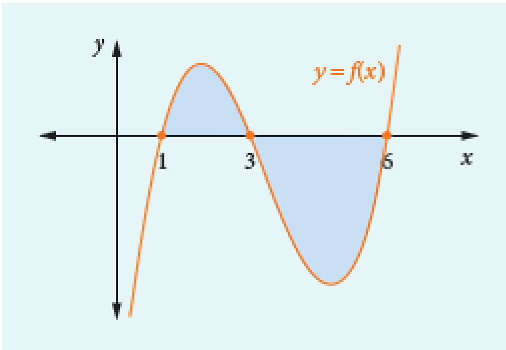
5 A $\int x^3(4x + 3)dx = \int (4x^4 + 3x^3)dx$

6 D $\int_0^{\frac{\pi}{6}} \cos(3x)\,dx = \left[\frac{\sin(3x)}{3}\right]_0^{\frac{\pi}{6}} = \frac{1}{3}\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) = \frac{1}{3}$

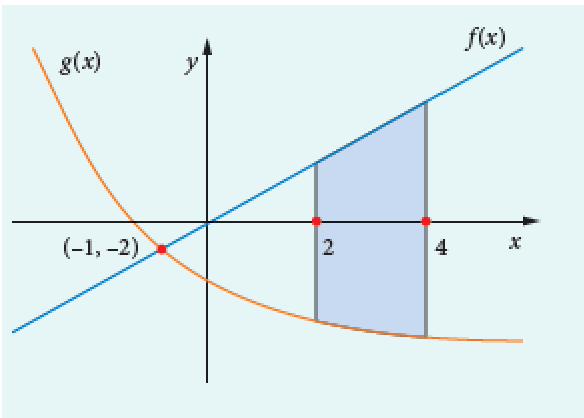
7 D $\int_{-3}^{2} f(x)\,dx$

8 C $\int_1^3 f(x)\,dx - \int_3^6 f(x)\,dx$

The shaded area between 3 and 6 is below the *x*-axis so the sign needs to be changed, and so it is subtracted.



9 B $\int_2^4 [f(x) - g(x)]dx$



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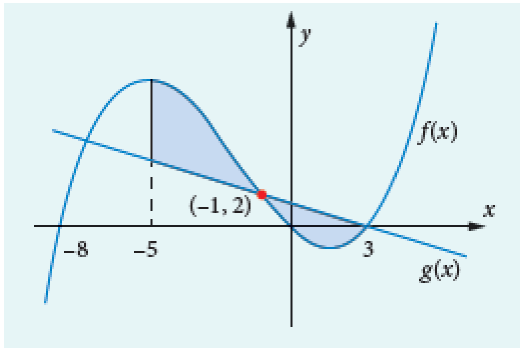
10 C $\int_{-5}^{-1}[f(x)-g(x)]dx + \int_{-1}^{3}[g(x)-f(x)]dx$



11 A $R'(t) = 100e^{-0.2t}$

$R(t) = \int_0^3 100e^{-0.2t}\,dt$

Short answer

12 **a** $\int (y^3 - 3y^2 + 4y + 1)dy = \frac{y^4}{4} - y^3 + 2y^2 + y + c$

b $\int -n^{-2}dn = \frac{-n^{-1}}{-1} = \frac{1}{n} + c$

c $\int \frac{-2}{x^2}dx = \frac{-2x^{-1}}{-1} + c = \frac{2}{x} + c$

d $\int (9x^2 - 2)(3x^3 - 2x + 4)dx = \frac{1}{2}(3x^3 - 2x + 4)^2 + c$

e $\int \sin(x)dx = -\cos(x) + c$

f $\int -3\cos(6x)dx = -\frac{3\sin(6x)}{6} + c = -\frac{\sin(6x)}{2} + c$

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g $\int -5\sin(10x)dx = \frac{5\cos(10x)}{10} + c = \frac{\cos(10x)}{2} + c$

h $\int e^{3t} dt = \frac{e^{3t}}{3} + c$

i $\int \frac{3}{e^{2x}} dx = \int 3e^{-2x} dx = \frac{3e^{-2x}}{-2} + c = -\frac{3}{2e^{2x}} + c$

j $\int 4\left(x-5\right)^{-3} dx = \frac{4\left(x-5\right)^{-2}}{-2} + c = -\frac{2}{\left(x-5\right)^{2}} + c$

k $\int \frac{1}{3(2x+7)^4} dx = \frac{1}{3}\int (2x+7)^{-4} dx = \frac{(2x+7)^{-3}}{-18} + c = -\frac{1}{18(2x+7)^3} + c$

l $\int \sqrt{4x+7} dx = \frac{2\left(4x+7\right)^{\frac{3}{2}}}{3\times 4} + c = \frac{\sqrt{\left(4x+7\right)^{3}}}{6} + c$

13 **a** $\int (x^4 + 7)\, dx = \frac{x^5}{5} + 7x + c$

b $\int (5x^4 - 2x^3 + 4x)\, dx = x^5 - \frac{x^4}{2} + 2x^2 + c$

c $\int (6x^3 - 8x^2 - 3) dx = \frac{3x^4}{2} - \frac{8x^3}{3} - 3x + c$

14 $\frac{dy}{dx} = 6x - 4$

$$y = \int 6x - \ 4\, dx$$
$$= 3x^2 - 4x + c$$
$$(-2, 22) \ \Rightarrow 22 = 12 + 8 + c$$
$$y = 3x^2 - 4x + 2$$

15 $f(x)=\int \frac{3}{2\sqrt{x}}\,dx=\frac{3}{2\times\frac{1}{2}}\sqrt{x}+c=3\sqrt{x}+c$

$f(x)=3\sqrt{x}+c$

$(1,\ 5)\ \Rightarrow\ 5=3\sqrt{1}+c$

$f(x)=3\sqrt{x}+2$

16 **a** $\int \frac{x^5-3x^3+7x}{x}\,dx=\int x^4-3x^2+7\,dx$

$$=\frac{x^5}{5}-x^3+7x+c$$

b $\int (2-3x)^2\,dx=\frac{(2-3x)^3}{3(-3)}+c=\frac{(2-3x)^3}{-9}+c$

c $\int \frac{3x^2-5x+2}{\sqrt{x}}dx=\int 3x^{\frac{3}{2}}-5x^{\frac{1}{2}}+2x^{-\frac{1}{2}}dx$

$$=2\times\frac{3x^{\frac{5}{2}}}{5}-2\times\frac{5x^{\frac{3}{2}}}{3}+2\times 2x^{\frac{1}{2}}+c$$

$$=\frac{6\sqrt{x^5}}{5}-\frac{10\sqrt{x^3}}{3}+4\sqrt{x}+c$$

17 **a** $\int_{-1}^{2}(12x^2-6x+1)\,dx=\left[4x^3-3x^2+x\right]_{-1}^{2}$

$$=\left(32-12+2\right)-\left(-4-3-1\right)$$

$$=30$$

b $\int_1^9 x^{\frac{1}{2}}dx=\left[\frac{2x^{\frac{3}{2}}}{3}\right]_1^9=\left[\frac{2\sqrt{x^3}}{3}\right]_1^9=18-\frac{2}{3}=17\frac{1}{3}$

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c $\int_5^{10}\frac{dx}{(x-4)^2}=\int_5^{10}(x-4)^{-2}dx=-\left[(x-4)^{-1}\right]_5^{10}=-\left[\frac{1}{(x-4)}\right]_5^{10}=-\left(\frac{1}{6}-1\right)=\frac{5}{6}$

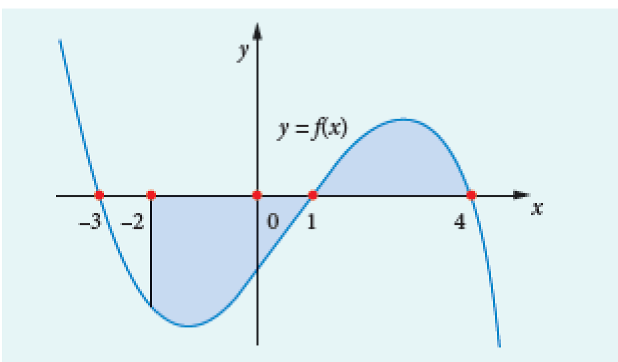
d $\int_0^{\frac{\pi}{3}}\sin(3x)dx=-\frac{1}{3}[\cos(3x)]_0^{\frac{\pi}{3}}=-\frac{1}{3}(\cos(\pi)-\cos(0))=-\frac{1}{3}(-1-1)=\frac{2}{3}$

e $\int_0^{\frac{\pi}{4}}\sin\left(2x+\frac{\pi}{3}\right)dx=-\frac{1}{2}\left[\cos\left(2x+\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{4}}$

$$=-\frac{1}{2}\left(\cos\left(\frac{5\pi}{6}\right)-\cos\left(\frac{\pi}{3}\right)\right)=-\frac{1}{2}\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}\right)=\frac{\sqrt{3}}{4}+\frac{1}{4}$$

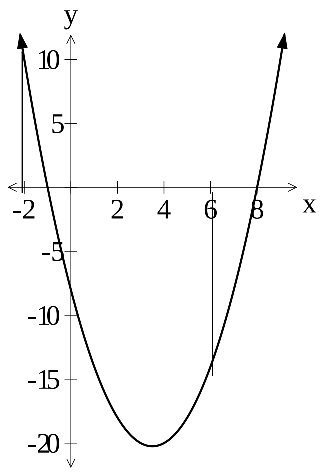
f $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}12\cos(3x)dx=\frac{12}{3}[\sin(3x)]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}=4(\sin(\pi)-\sin(-\pi))=0$

18



Area = $-\int_{-2}^{1}f(x)dx+\int_1^4 f(x)dx$

19 **a** $y = x^2 - 7x - 8$ from $x = -2$ to $x = 6$



x-intercept at $x = -1$

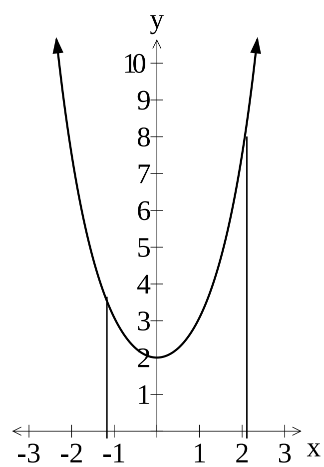
Area = $\int_{-2}^{-1}(x^2 - 7x - 8)\,dx - \int_{-1}^{6}(x^2 - 7x - 8)\,dx$

$$= \left[\frac{x^3}{3} - \frac{7x^2}{2} - 8x\right]_{-2}^{-1} - \left[\frac{x^3}{3} - \frac{7x^2}{2} - 8x\right]_{-1}^{6}$$

$$= \left(\frac{-1}{3} - \frac{7}{2} + 8\right) - \left(\frac{-8}{3} - \frac{28}{2} + 16\right) - \left\{\left(\frac{216}{3} - 63 - 48\right) - \left(\frac{-1}{3} - \frac{7}{2} + 8\right)\right\}$$

$$= 111$$

b $f(x) = e^{x} + e^{-x}$ from $x = -1$ and $x = 2$



x-intercept at $x = -1$

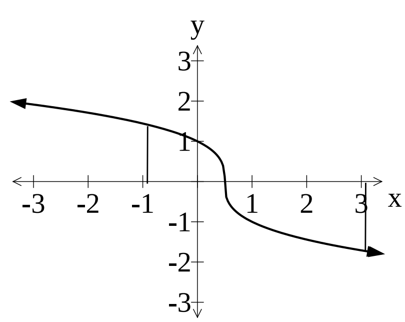
$$\text{Area} = \int_{-1}^{2} (e^{x} + e^{-x})\,dx = \left[e^{x} - e^{-x}\right]_{-1}^{2}$$

$$= \left[e \quad - e^{-x}\right]_{-1}^{2}$$

$$= \left(e^{2} - e^{-2}\right) - \left(e^{-1} - e^{1}\right)$$

$$= e^{2} + e - \frac{1}{e^{2}} - \frac{1}{e} \approx 9.6$$

c $y = (1-2x)^{\frac{1}{3}}$ from $x = -1$ to $x = 3$.



$$\text{Area} = \int_{-1}^{0.5} (1-2x)^{\frac{1}{3}}\,dx - \int_{0.5}^{3} (1-2x)^{\frac{1}{3}}\,dx = 4.83$$

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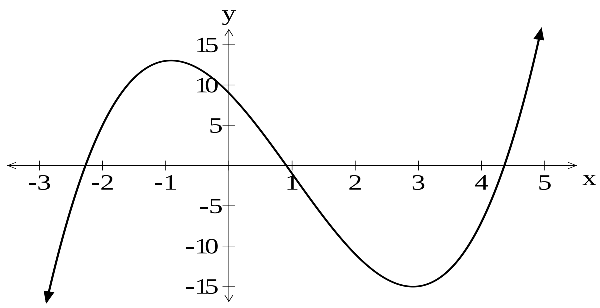
20 $y = x^3 - 3x^2 - 8x + 9$

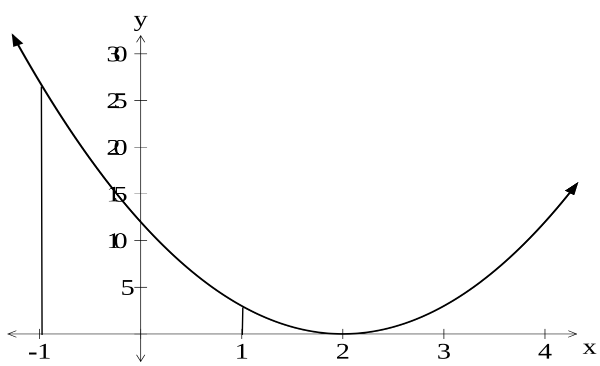


If $y = 0$, $x = ?$, $x = -2.27, 0.91, 4.36$

Area between function and the x-axis:

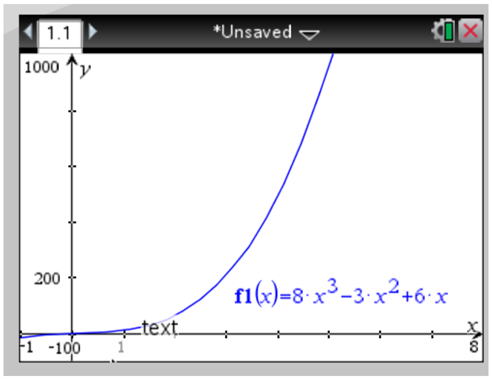
$$\int_{-2.27}^{0.91} x^3 - 3x^2 - 8x + 9\ dx - \int_{0.91}^{4.36} x^3 - 3x^2 - 8x + 9\, dx = 60.64 \text{ units}^2$$

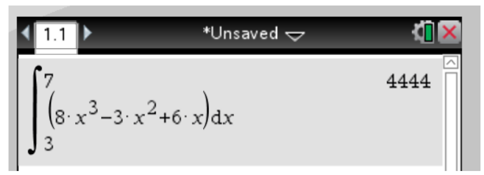
21 $f(x) = 3(x - 2)^2$, $x = -1$ and $x = 1$.



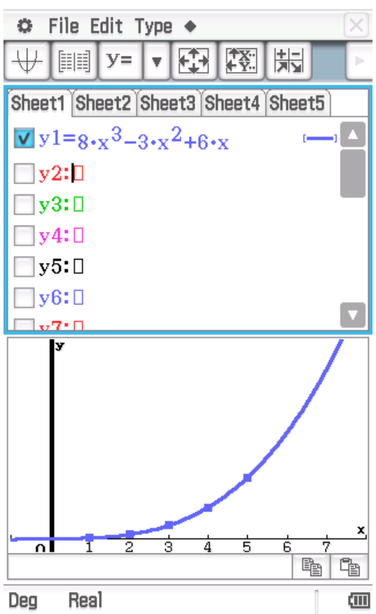
Area = $\int_{-1}^{1} 3(x-2)^2 dx = 26 \text{ units}^2$

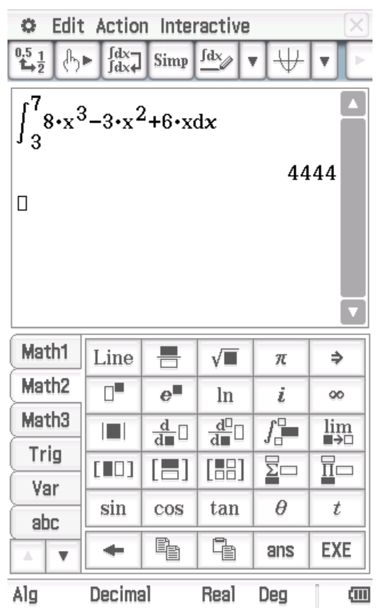
TI-Nspire CAS



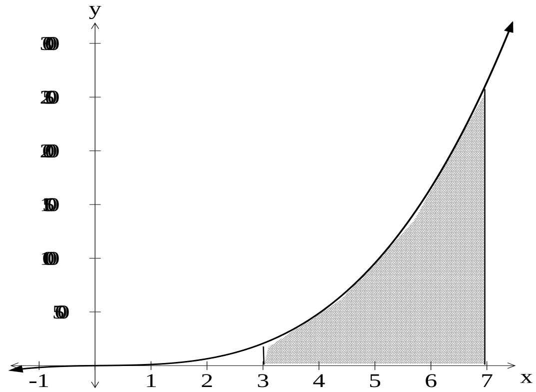


ClassPad



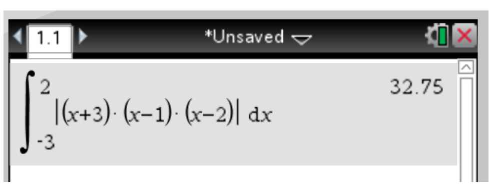


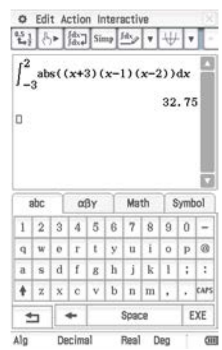
$y = 8x^3 - 3x^2 + 6x$ between $x = 3$ and $x = 7$.



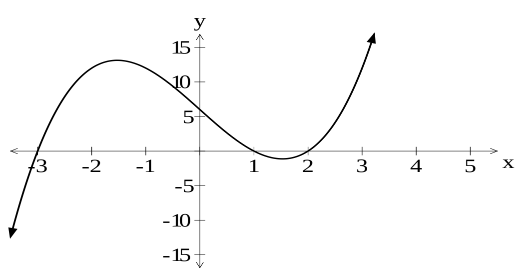
$$\text{Area} = \int_3^7 8x^3 - 3x^2 + 6x\,dx = 4\,444\ \text{units}^2$$

23 ClassPad TI-Nspire CAS





$y = (x + 3)(x - 1)(x - 2)$



Area between function and the x-axis:

$$\int_{-3}^{1} (x+3)(x-1)(x-2)\,dx - \int_{1}^{2} (x+3)(x-1)(x-2)\,dx = 32.75\ \text{units}^2$$

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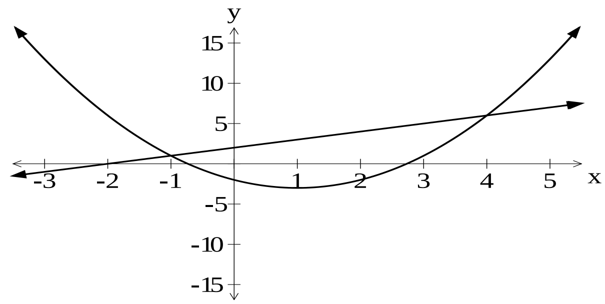
24 $y = x^2 - 2x - 2$ and $y = x + 2$

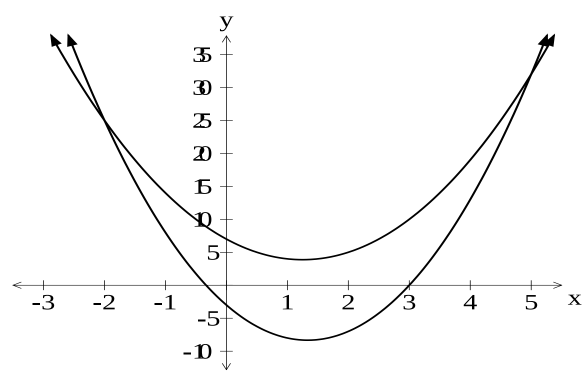


Points of intersection: (–1, 1) and (4, 6)

Area between the functions =

$$\int_{-1}^{4}(x+2)-(x^2-2x-2)\,dx = 20.8\overline{3} \text{ units}^2$$

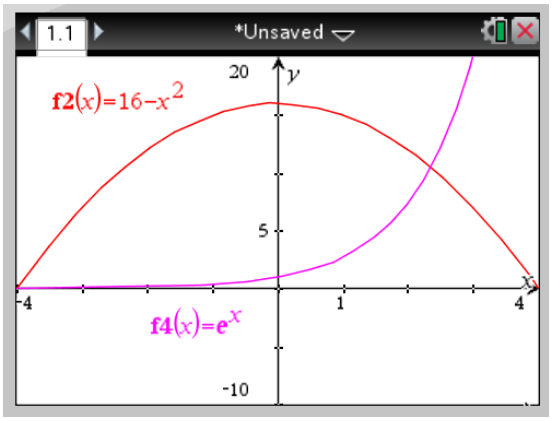
25 $y = 3x^2 - 8x - 3$ and $y = 2x^2 - 5x + 7$.

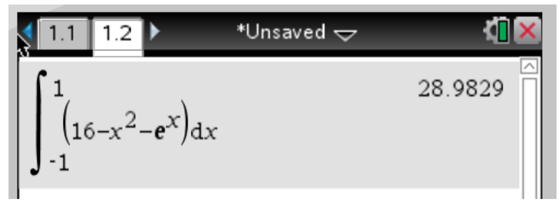


Points of intersection: $x = -2$ and $x = 5$.

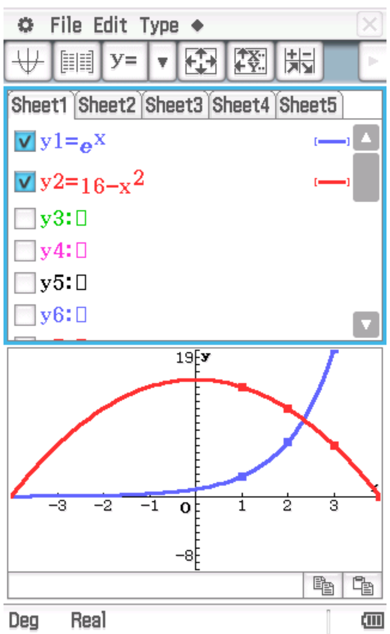
Area between the functions = $\int_{-2}^{5}(2x^2-5x+7)-(3x^2-8x-3)\,dx = 57.1\overline{6}$ units^2

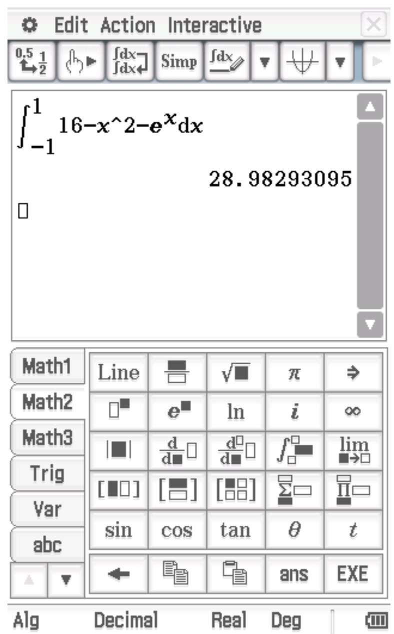
26 **TI-Nspire CAS**





ClassPad





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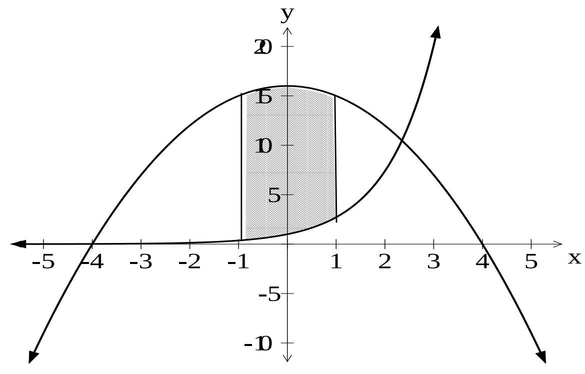
$f(x) = e^x$, $g(x) = 16 - x^2$ between $x = -1$ and $x = 1$



Area between the functions =

$$\int_{-1}^{1}\left(16 - x^2 - e^x\right)dx = 28.98 \text{ units}^2$$

27 $\int_0^5 C'(t)dt$ represents the total change in temperature of the liquid in the first 5 minutes.

28 $V'(t) = 150e^{-0.2t}$ litres/hour

a $V = \int_0^3 150e^{-0.2t}dt$

$$= \left[\frac{150e^{-0.2t}}{-0.2}\right]_0^3$$
$$= -750\left(e^{-0.6} - e^0\right)$$
$$= 338.4 \text{ L}$$

b $V = \int_3^6 150e^{-0.2t}dt = 185.71 \text{ L}$

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29 $P'(t) = \frac{t}{3} + 6$

a Total change in the population in the first 3 months

$$= \int_0^3 \frac{t}{3} + 6\, dt$$
$$= \left[\frac{t^2}{6} + 6t\right]_0^3$$
$$= \left(\frac{9}{6} + 18 - 0\right)$$
$$= 19.5$$

i.e. 1950 extra mice in the first 3 months.

b $42 = \int_0^t \frac{t}{3} + 6\, dt$

$$42 = \frac{t^2}{6} + 6t \ - 0$$
$$t^2 + 36t - 252 = 0$$
$$x > 0,\ \ x = 6$$

It will take six months for the population to reach 4200.

30 $R'(x) = 1500 - 3x^2 - 4x$

$$R(x) = \int 1500 - 3x^2 - 4x\, dx$$
$$= 1500x - x^3 - 2x^2 + c$$
$$R(0) = 0 \ \Rightarrow c = 0$$
$$R(x) = 1500x - x^3 - 2x^2$$
$$R(30) = \$16\ 200$$

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31 $a = 6t - 12$

$v_0 = 0$ m/s and $x_0 = -2$ m

$$
\begin{aligned}
v &= \int 6t - 12dt \\
&= 3t^2 - 12t + c \\
&\text{At } t = 0 \\
0 &= c \\
v &= 3t^2 - 12t \\
x &= \int 3t^2 - 12t\,dt \\
x &= t^3 - 6t^2 + c \\
&\text{At } t = 0 \\
-2 &= c \\
x &= t^3 - 6t^2 - 2 \\
x_5 &= 125 - 150 - 2 \\
&= -27 \text{ m}
\end{aligned}
$$

At $t = 5$ s, the particle is 27 m to the left of the initial position.

32 **a** $a = -9.8$ m/s^2

$v_0 = 30$ m/s and $x_0 = 0$ m

$$
\begin{aligned}
v &= \int -9.8dt \\
&= -9.8t + c \\
&\text{At } t = 0 \\
32 &= c \\
v &= -9.8t + 30 \\
x &= \int -9.8t + 30\,dt \\
x &= -4.9t^2 + 30t + c \\
&\text{At } t = 0 \\
0 &= c \\
x &= -4.9t^2 + 30t \\
x_4 &= 41.6 \text{ m}
\end{aligned}
$$

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b $v = -9.8t + 30$

$v_5 = -19\,\text{m/s}$

c Greatest height when $v = 0$.

$t = \frac{30}{9.8} = 3.06$

The object reaches its greatest height at $t = 3.06$ s.

33 $a = -20(1 + 2t)^2$ cm/s^2

$v_0 = 30$ cm/s

$$v = \int -20(1+2t)^2\,dt$$

$$= \frac{-20(1+2t)^3}{3\times 2} + c$$

$$v = \frac{-10(1+2t)^3}{3} + c$$

At $t = 0$,

$$30 = \frac{-10(1+2\times 0)^3}{3} + c$$

$$30 = \frac{-10}{3} + c$$

$$c = 33\frac{1}{3}$$

$$v = \frac{-10(1+2t)^3}{3} + \frac{100}{3}$$

$$v = \frac{100 - 10(1+2t)^3}{3}$$

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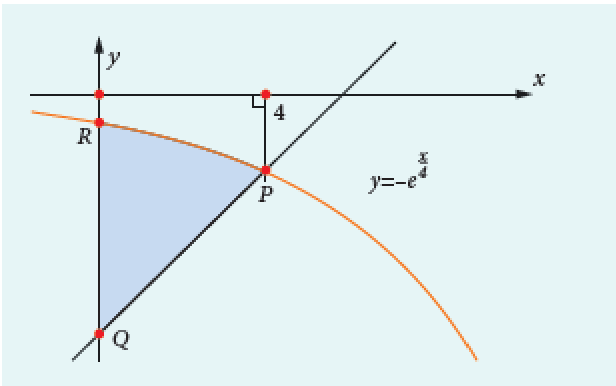
Application

34 $y = -e^{\frac{x}{4}}$

PQ is the perpendicular to the curve at the point *P*.



a $P(4, -e)$ as $x = 4$

b The equation of *PQ*:

$$\frac{dy}{dx} = -\frac{1}{4}e^{\frac{x}{4}}$$

At $x = 4$, $\frac{dy}{dx} = -\frac{e}{4}$

$$m_{\perp} = \frac{4}{e}$$

$$y = mx + b$$

$$-e = \frac{4}{e}(4) + b \Rightarrow b = -e - \frac{16}{e}$$

$$y = \frac{4}{e}x - e - \frac{16}{e}$$

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c If $x = 0$, $y = -e - \dfrac{16}{e}$

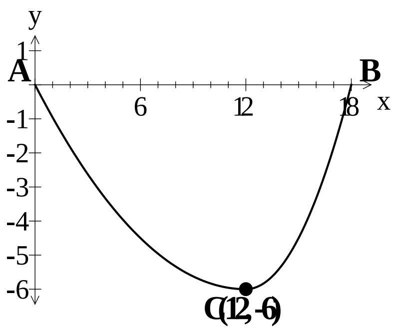
$\therefore Q\left(0, -e - \dfrac{16}{e}\right)$

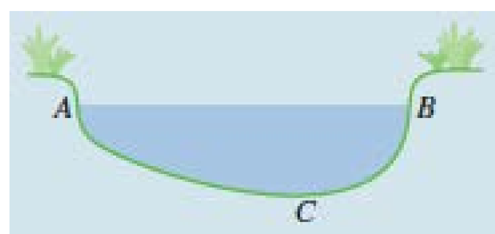
d $y = -e^{\frac{x}{4}}$. If $x = 0$, $y = -1$

$\therefore R(0, -1)$

e Area $= \int_0^4 -e^{\frac{x}{4}} dx - \int_0^4 \dfrac{4}{e}x - e - \dfrac{16}{e} dx = (-6.87) - (-22.65) = 15.78$

35 $y = \dfrac{x^2}{24} - x$





a Maximum depth of the river is 6 m. ($y = -6$ for either function at $x = 12$)

b Area $= \left|\int_0^{12} \dfrac{x^2}{24} - x\, dx\right| + \left|\int_{12}^{18} \dfrac{x^2}{6} - 4x + 18\, dx\right|$

$= 48 + 24$

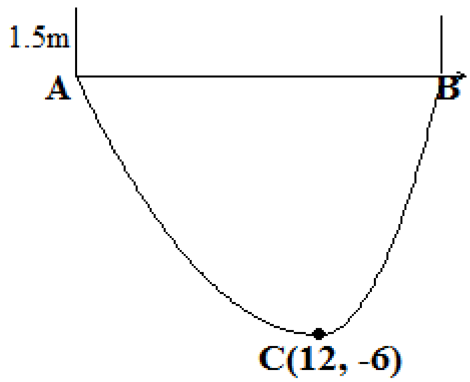
$= 72 \text{ m}^2$

c Volume = $72 \times 1.4 = 100.8$ m^3/s

d $\text{Volume}_{\text{day}} = 100.8 \times 60 \times 60 \times 24$ m^3/day

$\text{Volume}_{\text{day}} = 8\ 709\ 120$ m^3/day

e



Area between levees = 18 × 1.5 = 27

Flow rate in flood = (72 + 27) × 2.5 = 247.5 m^3/sec

Normal flow rate = 100.8 m^3/s

Ratio = $\dfrac{247.5}{100.8} = 2.46 : 1$

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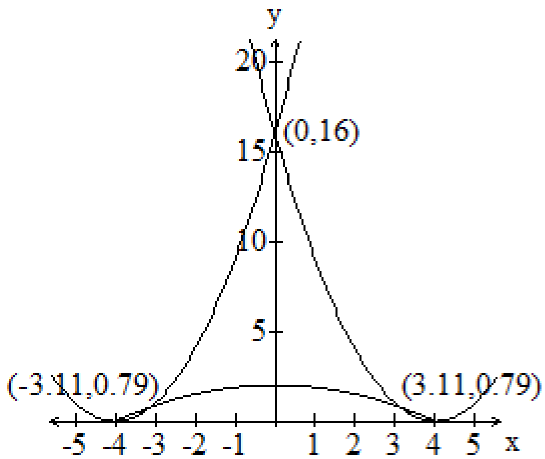
36 $y = x^2 - 8x + 16$, $y = x^2 + 8x + 16$ and $y = 2 - \dfrac{x^2}{8}$



$$\text{Area} = \int_{-3.11}^{0} x^2 + 8x + 16\,dx + \int_{0}^{3.11} x^2 - 8x + 16\,dx - \int_{-3.11}^{3.11} 2 - \frac{x^2}{8}\,dx$$

$$= \quad 21.10 \quad + \quad 21.10 \quad - \quad 9.935$$

$$= 32.26\,\text{m}^2$$

37 $a = 3t + 1$ m/s^2

At $t = 0$, $x = 0$ and $v = 15$ m/s

a $v = \dfrac{3t^2}{2} + t + c$

At $t = 0$, $15 = 0 + c$

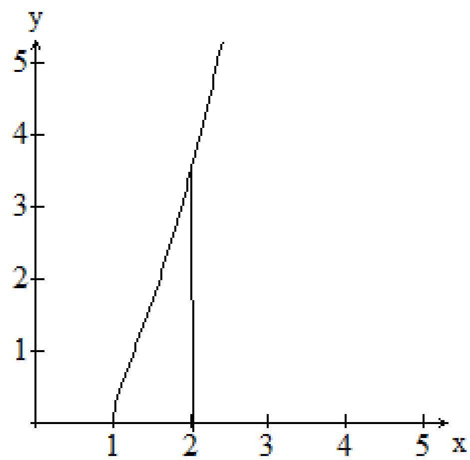
$v = \dfrac{3t^2}{2} + t + 15$ m/s

$v_3 = 31.5$ m/s

b $v = \dfrac{3t^2}{2} + t + 15$ m/s

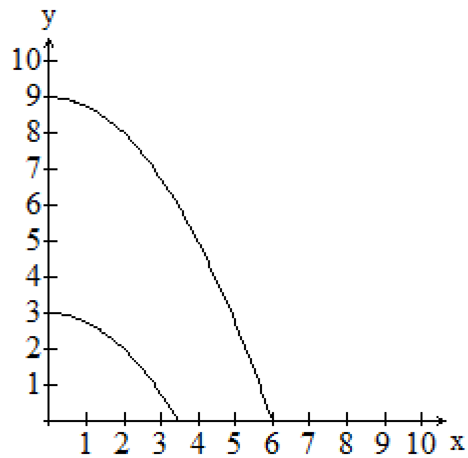
$\Delta = 1 - 4 \times 1.5 \times 15 = -89$, so there are no real zeros and the particle is never at rest.

38 $y = x\sqrt{x^2 - 1}$, the x-axis and the lines $x = 1$ and $x = 2$.



$$\text{Area} = \int_1^2 x\sqrt{x^2 - 1}\,dx$$
$$= 1.73 \text{ units}^2$$

39 $y = \dfrac{12 - x^2}{4}$ and $y = \dfrac{36 - x^2}{4}$

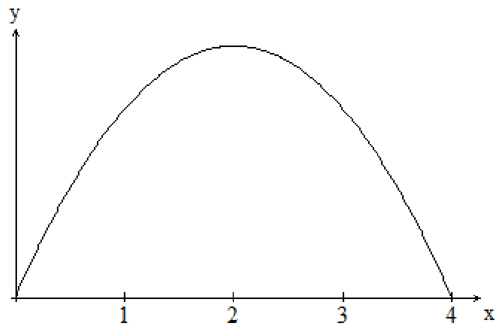


$\sqrt{12}$

$$\text{Area of driveway } = \int_0^6 \frac{36 - x^2}{4}\,dx - \int_0^{\sqrt{12}} \frac{12 - x^2}{4}\,dx$$
$$= (36 - 6.928...)$$
$$= 29.07...\,\text{m}^2$$

Cost of concrete = 29.07… × 0.1 × \$325 = \$944.83

40 $y = \dfrac{x(4-x)h}{400}$



$$\text{X-Area} = \int_0^4 \frac{x(4-x)h}{400}\,dx$$
$$= \frac{h}{400}\int_0^4 (4x - x^2)\,dx$$
$$= \frac{h}{400}\left[2x^2 - \frac{x^3}{3}\right]_0^4$$
$$= \frac{2h}{75}\ \text{m}^2$$

Volume = $6 \times \dfrac{2h}{75}$ m^3 = $0.16h$ m^3

Mass of water = $1000 \times 0.16h = 160h$ kg

Maximum sag is for 5 kg, so $5 = 160h$

Thus $h = 5 \div 160 = 0.031\,25$ m = 3.125 cm

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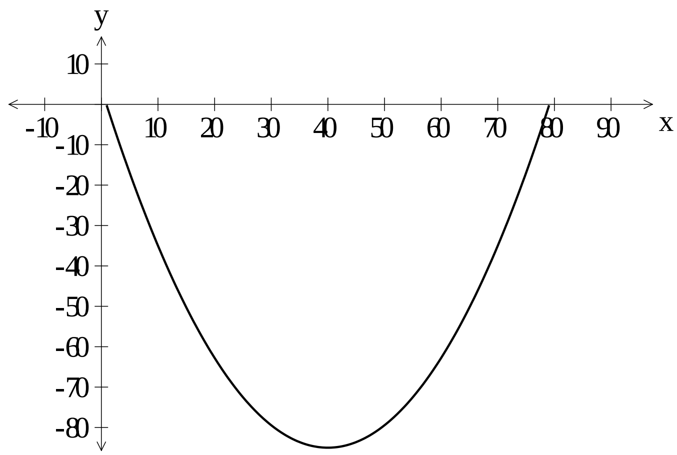
41 $y = \dfrac{5x^2 - 400x + 350}{90}$



x-intercepts are 0.885 and 79.115.

$$\text{Area } = \left| \int_{0.885}^{79.115} \frac{5x^2 - 400x + 350}{90} \, dx \right|$$

$$= \left| \frac{5}{90} \left[\frac{x^3}{3} - 40x^2 + 70x \right]_{0.885}^{79.115} \right|$$

$$= 4433 \, \text{cm}^2$$

$$\text{Volume} = \frac{4433}{100 \times 100} \times 3 \, \text{m}^3$$

$$= 1.3299 \text{ m}^3$$

$$\approx 13\,300 \, \text{L}$$

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